

Math 4111 Fall 2008
Exercises September 25

1. Let S be a subset of a metric space (X, d) . Show that

$$\partial(S) = \bar{S} \cap \overline{\mathbb{C}S}$$

2. Let S be a subset of a metric space (X, d) . Prove that

$$S^\circ = S \setminus \partial(S) = \mathbb{C}(\overline{\mathbb{C}S}).$$

3. Let S be a subset of a metric space (X, d) . Prove that

$$\mathbb{C}(\partial(S)) = S^\circ \cup (\mathbb{C}S)^\circ.$$

4. In a metric space (X, d) , the distance $d(p, A)$ of a point $p \in X$ to a nonempty subset A of X is defined by $d(p, A) = \inf(\{d(x, a) \mid a \in A\})$. Prove that $\overline{A} = \{x \in X \mid d(x, A) = 0\}$.

5. Prove that a vector subspace of an inner product space is closed.