

06E:204 Macroeconomics
Assignment 1

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September 1, 1999
Due: September 8, 1999

1. Consider the following representative agent model. The consumer has preferences given by

$$u(c, \ell) = \alpha c + \ell,$$

where c is consumption, ℓ is leisure, and $\alpha > 0$. The consumer is endowed with 1 unit of time and k_0 units of capital. The representative firm has a technology for producing consumption goods, given by

$$y = zk^\gamma n^{1-\gamma},$$

where z is total factor productivity, k is the capital input, n is the labor input, and $0 < \gamma < 1$. Let w denote the wage rate and r the rental rate on capital.

- (a) Solve for competitive equilibrium prices and quantities (hint: there will be 2 cases to consider here, i.e. the solution will be quite different depending on whether $\alpha z(1-\gamma)k_0^\gamma \leq 1$ or $\alpha z(1-\gamma)k_0^\gamma > 1$). Drawing a picture should help.
 - (b) What are the effects of an increase in z on equilibrium prices and quantities? Explain your results.
2. Consider an economy where the representative consumer has a utility function $u(c, \ell)$, where c is consumption and ℓ is leisure. The consumer has an endowment of 1 unit of time and k_0 units of capital. The representative firm has a production technology given by

$$y = f(k, n),$$

where k is the capital input and n is the labor input. The government imposes a proportional tax on labor income, where the tax rate is t , with $0 < t < 1$. Revenue from the tax on labor income is rebated to the representative consumer lump-sum. Let τ denote the lump sum transfer that the representative consumer receives.

- (a) Define a competitive equilibrium for this economy.
- (b) Show that the competitive equilibrium is not Pareto optimal.
- (c) How does the Pareto optimal allocation compare to the competitive equilibrium allocation? For example, is output larger or smaller, and is employment larger or smaller? Explain your results.