

06E:204 Macroeconomics
Assignment 2

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1. Consider the following representative agent model. The representative firm has access a technology which produces zn units of the consumption good for each unit of labor input, n , where $z > 0$. There is also a positive externality, in that the firm produces α units of good e for each unit of consumption goods produced, where $\alpha > 0$, but there is no mechanism which permits the firm to charge the consumer for producing e . For example, to produce consumption goods, the representative firm might grow trees, a byproduct of which is clean air. Clean air benefits consumers, but there is no market in clean air. The representative consumer's preferences are given by

$$u(c, l, e) = \log c + \beta \log l + \gamma \log e,$$

where $\beta > 0$, $\gamma > 0$, c is consumption, l is leisure, and e is the quantity of good e consumed.

- (a) Determine the socially optimal quantities of c , l , and e .
- (b) Determine consumption, leisure, e , and the real wage in a competitive equilibrium. How does the competitive equilibrium allocation differ from the social optimum? Explain any differences.
- (c) Now suppose that the government gives the representative producer a subsidy of s units of consumption goods for each unit of the consumption good produced. This subsidy is financed by a lump sum tax on the representative consumer. Determine consumption, leisure, e , and the real wage in a competitive equilibrium. Determine the value of s which achieves the social optimum, and explain your results.
- (d) Now, suppose that there is a mechanism by which the representative firm can charge a price for producing good e , and let p denote this price. Determine competitive equilibrium prices and quantities, and show that the competitive equilibrium allocation is Pareto optimal. Explain your results.

2. Suppose a representative agent economy where the representative consumer has preferences given by $U(c, l, g) = u(c, g) + v(l)$, where c is consumption, g is the quantity of the public good produced by the government, and l is leisure. Assume that $u(\cdot, \cdot)$ is strictly concave and that $v(\cdot)$ is strictly concave. The representative firm produces zn units of the consumption good for each n units of labor input. The government has a technology which can convert consumption goods one-for-one into public goods, and government purchases are financed by lump sum taxes on the consumer.
- (a) Suppose that g is set exogenously. Determine the effects of an increase in g on consumption, leisure, and the real wage in a competitive equilibrium, and explain your results.
- (b) Now, suppose that the government is benevolent, and sets g so as to maximize the representative consumer's welfare. In this case, determine the effects of an increase in z on consumption, leisure, the real wage, and g , in a competitive equilibrium. Explain your results.