1. Consider the following representative agent model. The representative firm has access to a technology which produces $zn$ units of the consumption good for each unit of labor input, $n$, where $z > 0$. There is also a positive externality, in that the firm produces $\alpha$ units of good $e$ for each unit of consumption goods produced, where $\alpha > 0$, but there is no mechanism which permits the firm to charge the consumer for producing $e$. For example, to produce consumption goods, the representative firm might grow trees, a byproduct of which is clean air. Clean air benefits consumers, but there is no market in clean air. The representative consumer’s preferences are given by

$$u(c, l, e) = \log c + \beta \log l + \gamma \log e,$$

where $\beta > 0$, $\gamma > 0$, $c$ is consumption, $l$ is leisure, and $e$ is the quantity of good $e$ consumed.

(a) Determine the socially optimal quantities of $c$, $l$, and $e$.

(b) Determine consumption, leisure, $e$, and the real wage in a competitive equilibrium. How does the competitive equilibrium allocation differ from the social optimum? Explain any differences.

(c) Now suppose that the government gives the representative producer a subsidy of $s$ units of consumption goods for each unit of the consumption good produced. This subsidy is financed by a lump sum tax on the representative consumer. Determine consumption, leisure, $e$, and the real wage in a competitive equilibrium. Determine the value of $s$ which achieves the social optimum, and explain your results.

(d) Now, suppose that there is a mechanism by which the representative firm can charge a price for producing good $e$, and let $p$ denote this price. Determine competitive equilibrium prices and quantities, and show that the competitive equilibrium allocation is Pareto optimal. Explain your results.
2. Suppose a representative agent economy where the representative consumer has preferences given by $U(c, l, g) = u(c, g) + v(l)$, where $c$ is consumption, $g$ is the quantity of the public good produced by the government, and $l$ is leisure. Assume that $u(\cdot, \cdot)$ is strictly concave and that $v(\cdot)$ is strictly concave. The representative firm produces $zn$ units of the consumption good for each $n$ units of labor input. The government has a technology which can convert consumption goods one-for-one into public goods, and government purchases are financed by lump sum taxes on the consumer.

(a) Suppose that $g$ is set exogenously. Determine the effects of an increase in $g$ on consumption, leisure, and the real wage in a competitive equilibrium, and explain your results.

(b) Now, suppose that the government is benevolent, and sets $g$ so as to maximize the representative consumer’s welfare. In this case, determine the effects of an increase in $z$ on consumption, leisure, the real wage, and $g$, in a competitive equilibrium. Explain your results.