1. Consider the following representative agent model. The representative consumer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t),$$

where $0 < \beta < 1$, $c_t$ is consumption, and $\ell_t$ is leisure. The consumer is endowed with one unit of time in each period, which can be allocated between work and leisure. The representative firm has access to a technology

$$y_t = z_t n_t,$$

where $y_t$ is output and $n_t$ is labor input. Assume that $z_t = z$, a constant, for all $t$. The government purchases $g$ units of the consumption good each period, and finances these purchases through lump-sum taxes and by issuing real bonds. The government’s budget constraint is

$$g + (1 + r_t)b_t = \tau_t + b_{t+1},$$

$t = 0, 1, 2, ...$, where $b_t$ is the quantity of government bonds that come due in period $t$, $r_t$ is the real interest rate on these bonds, and $\tau_t$ is the lump sum tax on the representative consumer. Assume that $b_0 = 0$.

(a) Suppose that $\tau_t = \tau^*$ for all $t$, and that $\beta = .99$. Determine $\tau^*$ in equilibrium.

(b) Alternatively, suppose that $\tau_0 = 1.1 \tau^*$, and $\tau_t = \tau^{**}$ for $t = 1, 2, 3, ...$. Determine $\tau^{**}$ in equilibrium. How does the change in tax policy from part (a) to part (b) affect equilibrium consumption, employment, and the real interest rate? Discuss your results.
2. Consider the same model as in question 1.

(a) Suppose that \( g_t = g > 0 \), and \( z_t = z > 0 \), for \( t = 0, 1, 2, \ldots \). Determine the real interest rate in periods \( t = 0, 1, 2, \ldots \).

(b) Suppose that \( g_t = g^* \) for \( t = 0, 1, 2, \ldots, T-1 \), and \( g_T = g^{**} > g^* \) for \( t \geq T \). Determine the path for consumption, employment, and the real interest rate for \( t \geq 0 \).

(c) Suppose alternatively that \( g_t = g^* \) for \( t = 0, 1, 2, \ldots, T-1, T+1, T+2, \ldots \), and \( g_T = g^{**} > g^* \). Determine the path for consumption, employment, and the real interest rate for \( t \geq 0 \).

(d) Explain your results in parts (a), (b), and (c).