

06E:204 Macroeconomics
Assignment 3

Steve Williamson
September 15, 1999
Due: September 22, 1999

1. Consider the following representative agent model. The representative consumer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t),$$

where $0 < \beta < 1$, c_t is consumption, and ℓ_t is leisure. The consumer is endowed with one unit of time in each period, which can be allocated between work and leisure. The representative firm has access to a technology

$$y_t = z_t n_t,$$

where y_t is output and n_t is labor input. Assume that $z_t = z$, a constant, for all t . The government purchases g units of the consumption good each period, and finances these purchases through lump-sum taxes and by issuing real bonds. The government's budget constraint is

$$g + (1 + r_t)b_t = \tau_t + b_{t+1},$$

$t = 0, 1, 2, \dots$, where b_t is the quantity of government bonds that come due in period t , r_t is the real interest rate on these bonds, and τ_t is the lump sum tax on the representative consumer. Assume that $b_0 = 0$.

- (a) Suppose that $\tau_t = \tau^*$ for all t , and that $\beta = .99$. Determine τ^* in equilibrium.
- (b) Alternatively, suppose that $\tau_0 = 1.1\tau^*$, and $\tau_t = \tau^{**}$ for $t = 1, 2, 3, \dots$. Determine τ^{**} in equilibrium. How does the change in tax policy from part (a) to part (b) affect equilibrium consumption, employment, and the real interest rate? Discuss your results.

2. Consider the same model as in question 1.
- (a) Suppose that $g_t = g > 0$, and $z_t = z > 0$, for $t = 0, 1, 2, \dots$. Determine the real interest rate in periods $t = 0, 1, 2, \dots$.
 - (b) Suppose that $g_t = g^*$ for $t = 0, 1, 2, \dots, T - 1$, and $g_t = g^{**} > g^*$ for $t \geq T$. Determine the path for consumption, employment, and the real interest rate for $t \geq 0$.
 - (c) Suppose alternatively that $g_t = g^*$ for $t = 0, 1, 2, \dots, T - 1, T + 1, T + 2, \dots$, and $g_T = g^{**} > g^*$. Determine the path for consumption, employment, and the real interest rate for $t \geq 0$.
 - (d) Explain your results in parts (a), (b), and (c).