

**6E:204 Macroeconomics  
Assignment 4**

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1. Consider an economy where the consumer values the stream of consumption according to

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where  $0 < \beta < 1$  and  $u(\cdot)$  is strictly increasing, twice differentiable, and strictly concave, with  $u'(0) = \infty$ . In period 0, there are  $k_0$  units of capital, and output is produced each period according to

$$y_t = \alpha k_t,$$

where  $y_t$  is output,  $\alpha > 0$ , and  $k_t$  is the capital stock. Capital depreciates at the rate  $\delta$ , where  $0 < \delta < 1$ .

- (a) Formulate the consumer's problem as a dynamic program. What is the state variable, and what are the choice or control variables?
- (b) Assuming the value function is differentiable and strictly concave, determine a condition involving  $\alpha$ ,  $\beta$ , and  $\delta$  which guarantees that consumption will increase over time, and interpret this condition.
2. Suppose an exogenous growth model, where the utility of the dynastic household is given by

$$\sum_{t=0}^{\infty} \beta^t N_t \ln c_t,$$

where  $0 < \beta < 1$ ,  $N_t$  is population, and  $c_t$  is per capita consumption. The population grows according to  $N_t = (1 + n)^t N_0$ , where  $n > -1$ , and each consumer has 1 unit endowment of labor time each period. Output is produced according to

$$Y_t = K_t^\alpha N_t^{1-\alpha},$$

where  $Y_t$  is aggregate output,  $K_t$  is the aggregate capital stock, and  $0 < \alpha < 1$ . Assume 100% depreciation.

- (a) Using guess-and-verify methods, determine the balanced growth path for this economy. What determines the growth rates of the aggregate capital stock, aggregate output, aggregate consumption, per capita output, and per capita consumption?
- (b) What are the effects of changes in  $\alpha$  and  $\beta$  on the capital/labor ratio, per capita output, and per capita consumption? Explain your results.