

6E:204 Macroeconomics
Assignment 5

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1. An economy consists of many agents, each of whom maximizes

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^\gamma}{\gamma}$$

where $0 < \beta < 1$, c_t is consumption, and $\gamma < 1$. Each agent has one unit of time available in each period, and can produce consumption goods according to

$$c_t = \alpha h_t^\theta \bar{h}_t^{1-\theta} u_t,$$

where $\alpha > 0$, h_t is the agent's human capital, u_t is time spent in consumption goods production by the agent, $0 < \theta < 1$, and \bar{h}_t denotes average human capital across all agents in the economy. That is, there is an externality; each agent is more productive the higher the human capital that other agents in the economy have. An individual agent accumulates human capital according to the production technology

$$h_{t+1} = \delta h_t (1 - u_t),$$

where $\delta > 0$. Assume that each agent treats $\{\bar{h}_t\}_{t=0}^{\infty}$ as being fixed (each agent is small relative to the population, so her decisions have a negligible effect on average human capital). Each agent has the same quantity of initial human capital, h_0 .

- (a) Set up the social planner's problem as a dynamic program, and solve this problem, noting that the social planner internalizes the externality and treats $\bar{h}_t = h_t$. You need to solve for the growth rates of human capital and consumption, and for u_t .
- (b) Solve for the competitive equilibrium, noting that there are no markets in which agents interact. The only interaction is through the human capital externality. Each agent maximizes utility given $\{\bar{h}_t\}_{t=0}^{\infty}$, and then in equilibrium $\{\bar{h}_t\}_{t=0}^{\infty} = \{h_t\}_{t=0}^{\infty}$. Solve for a balanced growth path where consumption and human capital grow at constant rates, and where u_t is constant.

- (c) How do the growth rates of consumption and human capital and u_t on the balanced growth path compare in parts (a) and (b)? Explain your results.