

6E:204 Macroeconomics
Assignment 6

STEVE WILLIAMSON

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1. Consider the following representative agent model. There is a representative consumer who has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t,$$

where $0 < \beta < 1$, and c_t is consumption. The consumer has one unit endowment of time available in each period. The production technology is given by

$$y_t = z_t k_t^\alpha n_t^{1-\alpha},$$

where y_t is output, k_t is the capital stock, n_t is employment, $0 < \gamma < 1$, and z_t is a technology shock. The log of z_t follows a first-order autoregressive process, that is

$$\ln z_{t+1} = \gamma \ln z_t + \epsilon_{t+1},$$

where ϵ_{t+1} is an i.i.d. random variable with $E[\epsilon_{t+1}] = 0$, and $0 < \gamma < 1$. The realization of ϵ_t is known at the beginning of period t . There is 100% depreciation of the capital stock in each period, so the aggregate resource constraint is given by

$$c_t + k_{t+1} = y_t.$$

- (a) Solve for a competitive equilibrium by solving the social planner's problem. That is, set up the social planner's problem as a dynamic program, explain what the state variables and choice variables are, and determine the optimal decision rules (hint: guess that the value function is linear in the log of k_t and the log of z_t).
- (b) Show that there are two propagation effects on output from the technology shock; one occurs through z_t , and the other through k_t . Explain.