1. Consider the following representative agent model. There is a representative consumer who has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t,$$

where $0 < \beta < 1$, and $c_t$ is consumption. The consumer has one unit endowment of time available in each period. The production technology is given by

$$y_t = z_t k_t^\alpha n_t^{1-\alpha},$$

where $y_t$ is output, $k_t$ is the capital stock, $n_t$ is employment, $0 < \gamma < 1$, and $z_t$ is a technology shock. The log of $z_t$ follows a first-order autoregressive process, that is

$$\ln z_{t+1} = \gamma \ln z_t + \epsilon_{t+1},$$

where $\epsilon_{t+1}$ is an i.i.d. random variable with $E[\epsilon_{t+1}] = 0$, and $0 < \gamma < 1$. The realization of $\epsilon_t$ is known at the beginning of period $t$. There is 100% depreciation of the capital stock in each period, so the aggregate resource constraint is given by

$$c_t + k_{t+1} = y_t.$$

(a) Solve for a competitive equilibrium by solving the social planner’s problem. That is, set up the social planner’s problem as a dynamic program, explain what the state variables and choice variables are, and determine the optimal decision rules (hint: guess that the value function is linear in the log of $k_t$ and the log of $z_t$).

(b) Show that there are two propagation effects on output from the technology shock; one occurs through $z_t$, and the other through $k_t$. Explain.