

6E:204 Macroeconomics
Assignment 7

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1. Consider the following one-sided search model. There is a continuum of agents who each maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $\beta = \frac{1}{1+r}$ is the discount factor, with $r > 0$, and $u(\cdot)$ has the same properties as in my notes. An unemployed agent receives an unemployment benefit of $b > 0$ at the beginning of each period, and then receives a wage offer. The wage offer is w_1 with probability π_1 and w_0 with probability $1 - \pi_1$, where $0 < w_0 < w_1$, and $0 < \pi_1 < 1$. All jobs have a separation rate of δ , where $0 < \delta < 1$. Employed workers can search on the job. When employed (in either the low-wage or the high-wage job), a worker receives a wage offer of w_1 with probability π_2 , and a wage offer of w_0 with probability $1 - \pi_2$.

- (a) Write down Bellman equations which determine the values of being unemployed and of holding low-wage and high-wage jobs, as of the end of the period.
 - (b) Determine conditions under which an unemployed worker will or will not accept a low-wage job.
 - (c) Determine how b , w_0 , w_1 , π_1 , and π_2 affect an unemployed worker's decision concerning whether or not to accept a low-wage job.
 - (d) Determine how the steady state unemployment rate depends on b .
2. Consider the following search model. Each agent in a continuum of agents with unit mass has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) - a_t],$$

where c_t is consumption and a_t is production. An unemployed agent is searching for a production opportunity, and receives such an opportunity with probability $\theta\gamma$, where $0 < \theta < 1$, and γ denotes the fraction of agents who are employed in the steady state. Here, there is a congestion externality, in that the probability of receiving a production opportunity decreases with the fraction of unemployed agents, $1 - \gamma$. To take advantage of a production opportunity, the unemployed agent must incur a cost $\alpha > 0$, in which case y units of output are produced, and the agent then becomes employed and must find a trading partner. For an employed agent, the probability of finding a trading partner is $b\gamma$, where $b > 0$.

- (a) Show that there are three steady state equilibria, and solve for π , γ , V_u , and V_e in each steady state equilibrium.
- (b) Determine how the congestion externality affects the unemployment rate, output, and welfare, i.e. compare your solution above with the one where production opportunities are received with probability θ .