

6E:204 Macroeconomics
Assignment 8

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November 17, 1998

Due: December 1, 1999

1. Consider a monetary search model which is identical to the one in my notes, except that there are two kinds of money. That is, in period 0 a fraction M_1 of agents are endowed with one unit each of type 1 money, and fraction M_2 are endowed with one unit each of type 2 money, where $0 < M_1 + M_2 < 1$. The two monies are distinguishable. For example they have different colors, but are otherwise identical in terms of their physical properties.
 - (a) Find all the steady state Nash equilibria, and determine conditions under which each equilibrium exists.
 - (b) Rank the equilibria in terms of welfare.
2. Consider the following search model. There is a continuum of agents with unit mass, each of whom has preferences given by

$$E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t u(c_t),$$

where c_t is consumption and $r > 0$. There are two types of agents denoted type 1 and type 2. Half of the population are type 1 agents, and half are type 2. At the end of each period, a type i agent can produce one unit of a type i good at no cost. In the following period, he/she meets some other agent at random. If the two agents both agree to trade, then they trade, but otherwise no trade takes place. An agent does not know the type of her would-be trading partner, and does not know the type of the would-be trading partner's good, at the time when trade takes place. If two agents trade, then they consume. When consumption takes place (and the type of the good becomes known), the trading partner has gone away and will never be seen again. A type i agent receives u_1 units of utility from consuming 1 unit of a type i good produced by someone else, receives u_2 if she consumes the good she produced, and receives u_3 if she consumes one unit of good $j \neq i$. Assume that $u_1 > u_2 > u_3$ and $\frac{u_1+u_3}{2} < u_2$. All goods produced

at the end of the previous period are perishable; they depreciate completely at the end of the current period. There are two kinds of money available at the first date. A fraction M of type 1 agents are endowed with blue money, and a fraction M of type 2 agents are endowed with pink money. Money comes in indivisible one-unit quantities, and no agent can hold more than one unit of some object at a given time.

- (a) Show that, if no one accepts money in a steady state equilibrium, then there is no trade.
- (b) Confine attention to steady state equilibria where everyone accepts both pink and blue money with probability one (so that both monies are essentially identical). Show that no such equilibria exist.
- (c) Now, suppose an equilibrium where type 1 agents accept blue money with probability one, but do not accept pink money, and where type 2 agents accept pink money with probability one, but do not accept blue money. Determine the value of holding money (note that everything is symmetric here, so the value of holding money is independent of its color given who will be holding it in equilibrium), the value of holding a good, and welfare in the steady state. Show that M needs to be chosen appropriately for this equilibrium to exist.
- (d) Explain your results in parts (a), (b), and (c).