

**6E:204 Macroeconomics**  
**Assignment 9**

STEVE WILLIAMSON

December 1, 1999

Due: December 8, 1999

1. Consider an overlapping generations model where there is a continuum of two-period-lived agents born in each period, having unit mass, each of whom has an endowment of  $y$  units of a perishable consumption good when young, and 0 units when old. There is a unit mass of one-period-lived old agents at  $t = 1$  who are collectively endowed with  $M_0$  units of fiat money. An agent born in period  $t$  has preferences given by

$$u(c_t^t, c_{t+1}^t) = \log c_t^t + \log c_{t+1}^t.$$

The government creates money in order to finance purchases of  $g$  units of consumption goods each period. That is, the government budget constraint is given by

$$p_t(M_t - M_{t-1}) = g,$$

for  $t = 1, 2, 3, \dots$ . Agents can store consumption goods from one period to the next, with each unit of consumption goods stored in period  $t$  yielding  $x$  units of consumption goods in period  $t + 1$ , where  $x > 1$ . However, there is a minimum scale to storage, in that storage yields nothing unless the quantity stored is greater than or equal to  $\gamma$ , where  $y > \gamma > \frac{y}{2}$ . Young agents must use the storage technology independently, i.e. storage cannot be shared. Let  $z$  denote the gross growth rate of the money supply, i.e.  $M_t = zM_{t-1}$  for all  $t$ , and note that  $z$  is an endogenous variable here. In a stationary monetary equilibrium, we will have  $\frac{p_{t+1}}{p_t} = \frac{1}{z}$ .

- (a) Show that it cannot be optimal for a young agent to hold money and use the storage technology in equilibrium (it will help to draw the agent's budget constraint and indifference curves).
- (b) Determine an agent's savings if he/she saves using money, and if he/she saves using the storage technology. Also, determine the agent's utility in each case.

- (c) Look for a stationary monetary equilibrium where agents are indifferent between holding money and using the storage technology, with the fraction of agents choosing each determined endogenously. Solve for  $z$ , the gross money growth rate, and  $\pi$ , the fraction of agents holding money in equilibrium (assume that parameters are such that this equilibrium exists).
- (d) Determine how  $z$  and  $\pi$  are affected by changes in  $x$  and  $\gamma$ , and explain your results.
2. Suppose a consumer that can borrow and lend at the one-period real interest rate  $r$  in each period, has initial assets  $A_0$ , and receives income  $w_t$  in period  $t$ , where  $t = 0, 1, 2, \dots$ . The consumer maximizes

$$-\sum_{t=0}^{\infty} \beta^t e^{-\alpha c_t},$$

where  $0 < \beta < 1$  and  $\alpha > 0$ . Show that the consumer's optimal consumption path has the property that the change in consumption from period  $t$  to period  $t + 1$  is a constant for all  $t$ .

3. Consider a representative agent economy where there is one productive unit which produces  $y_1$  units of the consumption good with probability  $\pi$ , and  $y_2$  units of the consumption good with probability  $1 - \pi$ , where  $0 < \pi < 1$ . There is one share outstanding in this single productive unit. In addition, there is a risky asset which pays off  $d_1$  units of the consumption good with probability  $\rho$  and  $d_2$  units with probability  $1 - \rho$ . The returns on this asset are uncorrelated with the returns on the productive unit. There are zero shares outstanding in this second asset (note that there is no production associated with the second asset, but the representative agent can buy and sell assets with this payoff structure; in equilibrium, the second asset will be priced such that the consumer will not want to buy or sell it). The representative consumer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t,$$

where  $0 < \beta < 1$ .

- (a) Determine the prices of each asset in each aggregate state (when output equals  $y_1, y_2$ ).
- (b) Determine the risk premium on each asset in each aggregate state, and explain your results.