

6E:204 Macroeconomics
Final Exam

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Instructions: Read the questions carefully and make sure to show all your work.
Good luck!

1. ($33\frac{1}{3}$ points) Consider the following search model. There is a continuum of agents with unit mass, each of whom has preferences given by

$$E_0 \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t u(c_t),$$

where c_t is consumption, $u(0) = 0$, and $r > 0$. A fraction M of agents is endowed with one unit each of indivisible fiat money in period 0. There is a continuum of commodities, and for each agent the fraction of these commodities which she likes is x , where $0 < x < 1$. If an agent likes a good, she receives utility $u^* > 0$ from consuming one unit of it. An agent receives zero utility from consuming goods she does not like. Each agent produces a good she does not like. Opportunities to produce goods arrive at random. In order to receive a production opportunity an agent cannot be holding money or a commodity. The probability that a production opportunity arrives in the current period (for an agent not holding anything in inventory) is θ , where $0 < \theta < 1$. If a production opportunity arrives, it costs $\gamma > 0$ units of utility to produce one indivisible unit of the commodity, and the commodity is then held in inventory without cost. No more than one unit of some object can be carried in inventory at a time. The agent need not accept a production opportunity when it arrives. Each agent is matched pairwise and at random with another agent each period.

- (a) Determine all the steady state Nash equilibria for this model. Carefully explain what economic forces give rise to multiple equilibria here.
- (b) Determine how M affects the existence of each steady state equilibrium, and explain your results.
2. ($33\frac{1}{3}$ points) Suppose an overlapping generations model where a continuum of two-period-lived agents with unit mass is born in period t . Each of these agents has preferences given by

$$u(c_t^t, c_{t+1}^t) = \ln c_t^t + \ln c_{t+1}^t,$$

where superscripts refer to an agent's date of birth and subscripts refer to the date when consumption takes place. Each young agent born in periods $t = 1, 2, 3, \dots$, has an endowment of y units of the perishable consumption good when young, and 0 units of the consumption good when old. In period 1, there is a continuum of one-period-lived old agents who are collectively endowed with M_0 units of fiat money. To consume when old, agents save in the form of fiat money and nominal government bonds. A nominal government bond is an asset which sells for q_t units of money in period t , and is a claim to one unit of money in period $t + 1$. Government bonds have minimum denominations, in that the real value of government bonds acquired by an individual in any period cannot be smaller than $\gamma > 0$. That is, if a young agent acquires b_t bonds in period t , then $p_t q_t b_t \geq \gamma$, where p_t is the price of fiat money in terms of consumption goods. Assume that $\gamma > \frac{y}{2}$, and that agents cannot share bonds. Let M_t denote the money supply in period t , and B_t the quantity of nominal government bonds issued in period t . Then, the government budget constraint is given by

$$p_t(M_t - M_{t-1}) + p_t q_t B_t = p_t B_{t-1},$$

for $t = 1, 2, \dots$, where $B_0 = 0$. That is, the value of money printed and bonds issued during the period is just sufficient to pay off the debt issued in the previous period. Assume that the government follows the financing rule

$$\frac{B_t}{M_t} = \alpha,$$

where $\alpha > 0$.

- (a) Look for a steady state equilibrium where $M_t = zM_{t-1}$ and $q_t = q < 1$, with z and q denoting positive constants, and solve for z and q . Determine the inflation rate and the nominal interest rate on bonds (the rate of return on bonds in units of money) in this steady state equilibrium. [Hint: Note that each young agent will be indifferent between holding government bonds and holding money in equilibrium; some will hold bonds, some will hold money.]
 - (b) What happens to the nominal interest rate and the inflation rate when α increases? Explain your results.
3. ($33\frac{1}{3}$ points) Suppose an environment where the representative consumer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $u(\cdot)$ is a strictly increasing and strictly concave utility function. There are two productive units in this economy, and a supply of one share in each productive unit, with shares traded on competitive markets each period. Let the price of a share in productive unit one be p_t and the price of a share in productive unit two be q_t . Each period, with probability π the output on productive unit one is y and the output on productive unit two is zero, and with probability $1 - \pi$ the output on productive unit one is 0 and the output on productive unit two is y . We have $0 < \pi < 1$.

- (a) Determine the equilibrium prices of the shares in productive units one and two.
- (b) Determine the equilibrium price of a risk-free asset, and find the risk premia on the shares in productive units one and two.
- (c) Explain your results.