

6E:204 Macroeconomics

Test 1

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Instructions: Read the questions carefully and make sure to show all your work.
Good luck!

- (33 $\frac{1}{3}$ points) Consider a one-period economy where the representative consumer has preferences given by the utility function $u(c, l)$, where c is consumption and l is leisure. The consumer has an endowment of 1 unit of time which can be allocated between work and leisure. The representative firm produces consumption goods according to $y = zn$, where y is output, z is the exogenous marginal product of labor, and n is labor input. The government imposes a proportional tax t on the firm's output, so that if the firm produces y units of consumption goods, it pays a tax ty in consumption goods to the government. The government rebates these tax revenues in a lump-sum fashion to the representative consumer.

- Show that the competitive equilibrium is not Pareto optimal.
- Determine the effects of an increase in t on output, labor supply, consumption, leisure, and the real wage, and explain your results.

- (33 $\frac{1}{3}$ points) Suppose a one-period economy where the representative consumer has preferences given by

$$u(c, l) = 2(c^{\frac{1}{2}} + l^{\frac{1}{2}}),$$

where c is consumption and l is leisure. The consumer has an endowment of 1 unit of time and k_0 units of capital. The representative firm has a technology given by

$$y = \min(zn, k),$$

where y is output of the consumption good, n is the labor input, k is the capital input, and $z > 0$.

- Solve for output, consumption, leisure, employment, and the real wage in a competitive equilibrium (a picture will help).

(b) What is the effect of an increase in z on output, consumption, leisure, the real wage, and employment? Explain your results.

3. ($33\frac{1}{3}$ points) Consider a representative agent economy where the representative consumer maximizes

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) + v(l_t) + u(g_t)],$$

where $0 < \beta < 1$, c_t is consumption, l_t is leisure, and g_t is the quantity of a public good. Assume that $u(\cdot)$ and $v(\cdot)$ are strictly increasing, strictly concave, with $u'(0) = v'(0) = \infty$, and $u'(\infty) = v'(\infty) = 0$. The consumer has one unit of time available in each period. The production technology is given by

$$y_t = n_t,$$

where y_t is output and n_t is labor input. The government has a technology which allows it to convert consumption goods one-for-one into public goods. The government budget constraint is

$$g_t + (1 + r_t)b_t = \tau_t + b_{t+1},$$

where b_{t+1} is the quantity of government bonds issued by the government in period t , with each of these bonds representing a promise to pay $1 + r_{t+1}$ units of the consumption good in period $t + 1$. Assume $b_0 = 0$. The representative consumer pays a lump-sum tax of τ_t in period t . Let w_t denote the wage rate in period t . In even periods, $t = 0, 2, 4, \dots$, the government sets $g_t = g^*$, where $g^* < \frac{y}{2}$, while in odd periods, $t = 1, 3, 5, \dots$, the government sets g_t so as to maximize the welfare of the representative consumer.

- (a) Show that output, consumption, the real wage, employment, and the real interest rate each follow a two-cycle in a competitive equilibrium. For example, output follows a sequence $\{y^*, y^{**}, y^*, y^{**}, \dots\}$.
- (b) Determine whether the real interest rate is higher in even periods or in odd periods, and explain your results (recall that the period t real interest rate is r_{t+1}).