

6E:204 Macroeconomics
Test 2

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Instructions: Read the questions carefully and make sure to show all your work.
Good luck!

1. ($33\frac{1}{3}$ points) Consider a model with exogenous growth where preferences of the dynastic household are given by

$$\sum_{t=0}^{\infty} \beta^t N_t \frac{c_t^\gamma}{\gamma},$$

where $0 < \beta < 1$, N_t is population, c_t is consumption, and $\gamma < 1$. The production technology is given by

$$Y_t = K_t^\alpha (A_t N_t)^{1-\alpha},$$

where $0 < \alpha < 1$, Y_t is aggregate output, K_t is the aggregate capital stock, and $A_t N_t$ is the labor input in efficiency units. There is 100% depreciation of the capital stock each period. Population grows at a constant rate, that is

$$N_t = (1 + n)^t N_0,$$

where $n > -1$, and A_t grows at a constant rate,

$$A_t = (1 + a)^t A_0,$$

where $a > -1$, and N_0 , A_0 , and K_0 are given. There is a government which acquires and destroys $gA_t N_t$ units of consumption goods each period, where $g > 0$. Suppose that the government taxes the dynastic household lump-sum to finance government purchases so that you can solve for a competitive equilibrium by solving the social planner's problem.

- (a) Set up the social planner's optimization problem as a dynamic programming problem.
- (b) Solve for a balanced growth path.

- (c) How does a change in g affect the growth rate of per-capita consumption, the growth rate of per-capita output, the level of per-capita consumption, the level of per-capita output, and the savings rate, on the balanced growth path. Explain your results.
2. ($33\frac{1}{3}$ points) The representative agent maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t + \alpha l_t),$$

where $0 < \beta < 1$, $\alpha > 0$, c_t is consumption, and l_t is leisure. The representative agent has one unit of time available in each period. Output is produced according to

$$y_t = z_t k_t^\alpha n_t^{1-\alpha},$$

where k_t is the capital input, and n_t is labor input, with $0 < \alpha < 1$. Capital depreciates by 100% each period. Assume that z_t is an i.i.d. random variable, and that k_0 is given.

- (a) Solve for consumption, investment, output, and employment as functions of the state variables in a competitive equilibrium. Assume interior solutions.
- (b) Can this model account for the fact that output and employment are positively correlated over the business cycle? Explain why or why not.
3. ($33\frac{1}{3}$ points) An economy consists of many agents, each of whom maximizes

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^\gamma}{\gamma}$$

where $0 < \beta < 1$, c_t is consumption, and $\gamma < 1$. If u_t units of time are spent by an agent in manufacturing, she produces $f(u_t)h_t$ consumption goods, where h_t is the agent's quantity of human capital. Assume that $f(0) = 0$, $f'(0) = \infty$, $f'(\infty) = 0$, and $f(\cdot)$ is strictly increasing and strictly concave. An agent can also spend time lobbying the government, and this can yield transfers from the government, which are financed by taxes on other agents. The quantity of net transfers that an agent receives, in units of consumption goods, is $g(x_t - \bar{x}_t)h_t$, where x_t is units of time spent in lobbying by the agent, and \bar{x}_t is average units of time spent lobbying by all agents. Assume that $g(0) = 0$, $g'(0) = \alpha > 0$, and that $g(\cdot)$ is strictly increasing. Thus, consumption for a given agent is given by

$$c_t = f(u_t)h_t + g(x_t - \bar{x}_t)h_t.$$

Each agent accumulates human capital according to

$$h_{t+1} = \delta h_t(1 - u_t - x_t),$$

where $\delta > 0$. Each agent has the same quantity of initial human capital, h_0 .

- (a) Set up a given agent's problem as a dynamic programming problem.
- (b) Find equations determining the growth rate of human capital, the growth rate of consumption, x_t , and u_t , on a balanced growth path. Note that you will not be able to solve explicitly for everything.
- (c) How does α affect the growth rate of human capital, the growth rate of consumption, x_t , and u_t on the balanced growth path? Explain your results.