1. Consider the following monetary search model. There is a continuum of agents, and each agent has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t).$$

Half of the population consists of type 1 agents, and the other half are type 2 agents. Each period, every agent is matched at random with another agent. The probability that an agent meets with someone of his or her own type is $\alpha$, and he or she meets the other type with probability $1 - \alpha$, where $0 < \alpha < 1$. An agent can produce one unit of an indivisible good at the end of each period at no cost. In period 0, a fraction $M$ of agents of each type are endowed with one unit each of indivisible fiat money. Assume free disposal. An agent can hold at most one unit of some object at a time, and cannot simultaneously hold goods and money. An agent cannot consume the good that he or she produces. However, a good produced by another agent of the same type can always be consumed, yielding utility $u^*$. There is a probability $x$ that the good produced by an agent of the other type will be something that an agent will like. Therefore, if a type $i$ agent meets a type $j$ agent, where $j \neq i$, then there is a probability $x$ that the type $j$ agent could produce a good that the type $i$ agent would like. If the agent likes the other agent’s good, consumption of the good will yield utility $u^*$, but if the agent does not like the good, then it would yield zero utility.

(a) Determine all of the steady state Nash equilibria.

(b) Rank these equilibria in terms of welfare and explain your results. In particular, how do $\alpha$ and $x$ matter for your welfare results?

2. Consider a representative agent economy where the representative consumer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t.$$
In this economy there are two productive technologies. Technology 1 produces \( \theta_i y_i \) units of consumption goods in state \( i \), for \( i = 1, 2 \), while technology 2 produces \( (1 - \theta_i)y_i \) units of consumption goods in state \( i \), \( i = 1, 2 \). Assume that \( y_1 > y_2 \), \( \theta_1y_1 > \theta_2y_2 \), and \( (1 - \theta_1)y_1 < (1 - \theta_2)y_2 \). Let \( s_t \) denote the aggregate state, where \( s_t \in \{0, 1\} \). Assume that \( \Pr[s_t = i \mid s_{t-1} = i] = \pi \), for \( i = 1, 2 \), where \( \pi > \frac{1}{2} \). In this economy, there are two assets traded: shares in technologies 1 and 2. There is a supply of one share in each technology in existence, and ownership of each share is a proportional claim on the output from that technology.

(a) Determine the prices of each share in each state of the world. Do share prices move procyclically or countercyclically?