6E:204 Macroeconomics
Assignment 3

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1. Consider the following overlapping generations growth model. Time is indexed by $t = 0, 1, 2, ...$, and in period t there are L_t two-period-lived consumers born, where $L_t = (1+n)^t L_0$, with L_0 given and $n > 0$. At $t = 0$ there are some one-period-lived old agents who are collectively endowed with K_0 units of capital, and who maximize consumption at $t = 0$. A consumer born in period t has preferences given by

$$u(c_t^y, c_{t+1}^o) = 2(c_t^y)^{\frac{1}{2}} + 2(c_{t+1}^o)^{\frac{1}{2}},$$

where c_t^y (c_t^o) denotes consumption of a young (old) consumer in period t. Each consumer has one unit of time available when young. The production technology is given by

$$Y_t = K_t^\alpha L_t^{1-\alpha},$$

where Y_t is output, K_t is the capital input, and L_t is the labor input. One unit of the consumption good can be converted into one unit of capital, and vice-versa. Let τ_t^y (τ_t^o) denote the lump-sum tax paid by a young (old) agent in period t, and let w_t denote the wage rate and r_t the rental rate on capital.

   (a) Determine the capital/labor ratio, consumption of the young, and consumption of the old, in a socially optimal steady state.

   (b) Consider the following "pay-as-you-go" social security system. The government sets $\tau_t^y = \tau$ for all t, and sets taxes for old consumers in each period so that the government budget balances.

      i. Write down the government budget constraint, and determine τ_t^o.

      ii. Show how a change in τ affects the steady state capital/labor ratio.

      iii. Show that it is possible, if τ is set correctly, to achieve the socially optimal steady state as a long-run competitive equilibrium.

(c) Consider a "fully-funded" social security system. Here, the government uses taxes on young consumers to finance the acquisition of government capital. That is, in period t the government acquires K^g_{t+1} units of capital, then in period $t+1$ the government rents this capital to firms, and then converts it to consumption goods, and then transfers the proceeds lump sum in equal amounts to old consumers. We then have $K^g_{t+1} = L_t \tau^y_t$ and $K^g_{t+1}(1 + r_{t+1}) = -L_t \tau^o_{t+1}$. Suppose that $K^g_{t+1} = L_t g$ for all t, where g is a constant.

i. Write down the government budget constraint.
ii. Show how a change in g affects the steady state capital/labor ratio.
iii. Is it possible to set g in the appropriate way to achieve the socially optimal steady state as a long-run competitive equilibrium? Why or why not?
iv. What do your results have to say about the efficacy of pay-as-you-go vs. fully-funded social security schemes?