

6E:204 Macroeconomics
Assignment 5

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1. Consider an economy where the representative consumer values the streams of consumption and leisure according to

$$\sum_{t=0}^{\infty} \beta^t [\ln c_t + \gamma \ln l_t],$$

where $0 < \beta < 1$. The consumer has 1 unit of time available in each period to allocate between work and leisure. In period 0, there are k_0 units of capital, and output is produced each period according to

$$y_t = \delta k_t^\alpha n_t^{1-\alpha},$$

where y_t is output, $\delta > 0$, k_t is the capital stock, n_t is the labor input, and $0 < \alpha < 1$. There is 100% depreciation of capital each period.

- (a) Formulate the consumer's problem as a dynamic program. What is the state variable, and what are the choice or control variables?
 - (b) Using a guess-and-verify approach, determine the decision rules for investment, consumption, and leisure, and determine the steady state levels of the capital stock, consumption, output, and leisure.
 - (c) Explain your results.
2. Consider the following consumption/storage problem. A consumer initially has k_0 units of consumption goods. In even periods, $t = 0, 2, 4, \dots$, if α_1 consumption goods are stored in period t , then one unit of consumption is the output in period $t + 1$. However, in odd periods, $t = 1, 3, 5, \dots$, if α_2 units of consumption goods are stored in period t , then one unit of consumption is the output in period $t + 1$. The consumer's preferences are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $u(\cdot)$ is strictly increasing and strictly concave. Assume that $\alpha_1 < \beta < \alpha_2$ and $\alpha_1 \alpha_2 < \beta^2$.

- (a) Write down the consumer's problem as a dynamic programming problem. Note that the value function will be different in even and odd periods. That is, we can formulate the problem in terms of two Bellman equations, with $v_e(k_t)$ denoting the value of the problem in even periods and $v_o(k_t)$ denoting the value of the problem in odd periods.
- (b) Assume that the value functions are strictly concave and differentiable, and show that consumption increases from t to $t + 1$ if t is even, and decreases if t is odd. Also show that the trend in consumption is negative, i.e. consumption decreases between t and $t + 2$ for any t .
- (c) Explain your results.