

**6E:204 Macroeconomics**  
**Assignment 7**

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1. Suppose a representative agent economy where the representative consumer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t \ln c_t,$$

and the production technology is given by

$$y_t = z_t k_t^\alpha n_t^{1-\alpha},$$

where  $0 < \alpha < 1$ ,  $z_t$  is a technology shock,  $k_t$  is the capital stock, and  $n_t$  is the labor input. There is 100% depreciation of the capital stock each period. The consumer has one unit of time available each period. The log of the technology shock follows an  $AR(1)$  process, that is

$$\ln z_{t+1} = \rho \ln z_t + \varepsilon_{t+1},$$

where  $0 < \rho < 1$ , and  $\varepsilon_{t+1}$  is an i.i.d. random variable.

- (a) Determine equilibrium consumption, output, and investment.
  - (b) Show that the  $\ln k_{t+1}$  can be written as a linear function of  $\varepsilon_{t-i}$ ,  $i = 0, 1, 2, \dots$ , and explain how and why this relationship depends on  $\alpha$  and  $\rho$ .
2. Suppose a consumer who has preferences given by

$$E_0 \sum_{t=0}^{\infty} \frac{c_t^{1-\gamma}}{1-\gamma},$$

for  $\gamma > 0$ . At the beginning of period  $t$ , the consumer has assets  $A_t$ , and chooses current consumption  $c_t$ . The gross rate of return on assets held from the end of period  $t$  until the beginning of period  $t+1$  is  $R_{t+1}$ . That is, assets at the beginning of period  $t+1$  will be  $A_{t+1} = R_{t+1}(A_t - c_t)$ . However,  $R_{t+1}$  is unknown when the period  $t$  consumption decision is made. Assume that  $R_{t+1}$  is an i.i.d.

random variable. Guess that the value function takes the form  $V(A_t) = \frac{BA_t^{1-\gamma}}{1-\gamma}$ , where  $B > 0$ , and determine the optimal decision rule for consumption. See if you can solve for  $B$ , but at the minimum, find an equation that determines  $B$ , and comment on the determinants of  $B$ , and how this affects the decision rule for  $c_t$ .