6E:204 Macroeconomics
Assignment 9

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1. Consider an endogenous growth model where the representative household maximizes

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^\gamma}{\gamma},$$

where $0 < \beta < 1$, c_t is consumption, and $\gamma < 1$. Each agent has one unit of time available in each period, and can produce consumption goods according to

$$c_t = \alpha h_t u_t,$$

where $\alpha > 0$, h_t is the agent's human capital, and u_t is time spent in consumption goods production by the agent. The government makes decisions about how much public education is provided, and we will model this as if the government can force the representative household to spend v_t units of time in period t providing public education. The technology for accumulating human capital is given by

$$h_{t+1} = v_t h_t (1 - u_t - v_t).$$

Assume that the government sets $v_t = v$, a constant, for all t.

   (a) Given v, determine the growth rates of human capital and consumption in a competitive equilibrium, and also determine u_t.

   (b) Suppose that the government chooses v to maximize the growth rate of consumption. What is the optimal v?

   (c) Determine the value of v which will maximize the welfare of the representative household. Is this the same as the value of v which maximizes the growth rate of consumption, from part (b)? Explain your results.

2. Consider the following search model. Each agent in a continuum of agents with unit mass has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - a_t \right],$$

where c_t is consumption and a_t is production. The unemployed are searching for a production opportunity, and receive one production opportunity each period. Each unemployed person receives one production opportunity each period, and all production opportunities have a cost of α^*. Agents are of two types, with half the population being type 1 and half the population being type 2. When a type i agent receives a production opportunity, he or she produces 1 unit of the type i good, for $i = 1, 2$. Once an agent produces, he or she then searches for a trading partner. Type 1 agents receive u_1 units of utility if they consume a type 1 good, and u_2 units of utility if they consume a type 2 good, while type 2 agents receive u_1 units if they consume a type 2 good and u_2 units if they consume a type 1 good. Assume that $u_1 > u_2 > \alpha^*$. Let γ_i denote the fraction of agents who are type i and employed, let π_i denote the probability that a type i agent accepts a production opportunity, and let δ_i denote the probability that a type i employed agent accepts a type j good, where $j \neq i$. When employed, an agent meets a type i agent with probability γ_i, for $i = 1, 2$. Confine attention to symmetric pure strategy steady state equilibria, i.e. equilibria where $\pi_1 = \pi_2 = \pi$, $\delta_1 = \delta_2 = \delta$, and $\gamma_1 = \gamma_2 = \gamma$, with $\pi \in \{0, 1\}$ and $\delta \in \{0, 1\}$.

   (a) Solve for the steady state equilibria (there are four).

   (b) Can these equilibria be Pareto ranked? If so, rank them and explain your results.