

6E:204 Macroeconomics
Final Exam

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Instructions: Read the questions carefully and make sure to show all your work.
Good luck!

1. ($33\frac{1}{3}$ points) Consider the following representative agent model. The representative agent has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t,$$

where $0 < \beta < 1$, and c_t is consumption. The representative agent has one unit of time each period, and the production technology is given by

$$y_t = k_t^\alpha n_t^{1-\alpha},$$

where y_t is output of consumption goods, k_t is the capital stock, n_t is the labor input, and $0 < \alpha < 1$. Capital depreciates by 100% each period. In period t , it requires z_t units of the consumption good to produce one unit of capital, which then becomes productive in period $t + 1$. Assume that z_t is an i.i.d. random variable.

- (a) Determine output, consumption, and investment in a competitive equilibrium.
 - (b) In the data, output, consumption, and investment are mutually positively correlated. Is this true in the model? Explain why or why not.
2. ($33\frac{1}{3}$ points) Consider the following search model. There is a continuum of agents, each having preferences given by

$$E_0 \sum_{t=0}^{\infty} \frac{1}{1+r} \ln [u(c_t) - a_t],$$

where $r > 0$, $u(\cdot)$ is an increasing function with $u(1) = u^*$ and $u(0) = 0$, c_t is consumption, and a_t denotes costs of production. In period 0, a fraction M

of agents is endowed with one unit each of indivisible fiat money. Assume free disposal. There are n types of agents, indexed by $i = 1, 2, \dots, n$. Each agent is capable of producing one unit of an indivisible good at no cost at the end of any period. This good can be stored costlessly and indefinitely. An agent can store only one object (a good or money) at a time. A type i agent consumes only goods produced by a type $i + 1$ agent, for $i = 1, 2, \dots, n - 1$. A type n agent consumes only the goods produced by a type 1 agent. The fraction of type i agents in the population is $\frac{1}{n}$ for $i = 1, 2, \dots, n$. Assume that $n \geq 3$. Each period, agents are matched bilaterally and at random.

- (a) Determine all the steady state Nash equilibria for this model. Determine welfare in each of these equilibria, and explain your results.
 - (b) Now, suppose that no agents are endowed with fiat money in the first period, and that the model is otherwise identical, except for the following. In the first period, an agent can, at a cost $\gamma \geq 0$, produce one unit of an indivisible and nonperishable object that no one wishes to consume. Think of this object as privately-produced money. Again, agents can store at most one unit of some object in any period.
 - i. Determine all of the steady state Nash equilibria.
 - ii. In the equilibrium where private money is produced and accepted (note that this requires that agents be indifferent between producing money and a good in the first period) determine how the quantity of private money in circulation and welfare varies with γ , and explain your results.
3. ($33\frac{1}{3}$ points) Consider a representative agent economy where the representative agent has preferences given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $0 < \beta < 1$, c_t is consumption, and $u(\cdot)$ is an increasing and strictly concave function. Aggregate output, y_t , is the exogenous output of a single firm. There is a single divisible share in this firm which is traded on a competitive market in each period. We will let asset 0 denote this share in aggregate output. Assume that y_t is an i.i.d. random variable. There are also n other assets which are in zero net supply. These assets are indexed by $i = 1, 2, \dots, n$, and the i^{th} asset is an i -period real bond. That is, in period t , asset i is a claim to one unit of consumption in period $t + i$, for $i = 1, 2, \dots, n$. Each asset is traded competitively until it matures, i.e. until the payoff is received. Let p_t^i denote the price of asset i in period t .

- (a) Determine the price of asset 1 as a function of current output, y , and let $p^1(y)$ denote this price.
- (b) Now, given $p^1(y)$, determine $p^2(y)$ the price of asset 2, and derive a general formula for $p^i(y)$, the price of asset i .
- (c) The yield to maturity on asset i is given by

$$r^i(y) = \frac{\bar{A}^i}{p^i(y)} - 1,$$

and the yield curve is a plot of the yield to maturity as a function of time to maturity, i.e. a plot of $r^i(y)$ as a function of i . The expectations theory of the term structure of interest rates predicts that the yield curve is upward-sloping when short-term interest rates are expected to rise, and that it is downward-sloping when short-term interest rates are expected to fall.

- i. Show how the level of output determines whether the yield curve is upward-sloping or downward-sloping, and explain your results.
- ii. Is the model consistent with the expectations theory? Explain why or why not.