

6E:204 Macroeconomics
Test 1

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September 28, 2000

Instructions: Read the questions carefully and make sure to show all your work.
Good luck!

- (33 $\frac{1}{3}$ points) Consider a one-period economy where the representative consumer has preferences given by the utility function $u(c, l)$, where c is consumption and l is leisure. The consumer has an endowment of 1 unit of time which can be allocated between work and leisure. The representative firm produces consumption goods according to $y = n$, where y is output and n is labor input. The government purchases an exogenous quantity of the consumption good, g , and finances this expenditure by imposing a proportional tax t on the consumer's labor income. That is, the consumer's after-tax wage income is $(1 - t)w(1 - l)$, where t is the tax rate and w is the real wage rate.
 - Write down the government's budget constraint.
 - Is the competitive equilibrium Pareto optimal? If it is, show why, and if it is not, show why not.
 - Determine the effects of an increase in g on labor supply, and explain why labor supply could increase or decrease when g increases.
- (33 $\frac{1}{3}$ points) Suppose a one-period economy where the representative consumer has preferences given by

$$U(c, l, x) = u(c) + v(l) - s(x)$$

where c is consumption, l is leisure, and x is the quantity of pollution produced by the representative firm. Assume that $u(\cdot)$ and $v(\cdot)$ are strictly increasing and strictly concave, and that $s(\cdot)$ is strictly increasing and strictly convex. The representative firm has a linear technology given by

$$y = zn,$$

where $z > 0$, y is output of consumption goods, and n is labor input. In producing output, the firm also produces pollution; that is, αn units of pollution are produced if n units of labor are used in production, where $\alpha > 0$.

- (a) Determine the Pareto optimal allocation.
- (b) Show that a competitive equilibrium is not Pareto optimal.
- (c) Suppose that the government requires the firm to pay a tax t per unit of pollution that it produces. The government takes the taxes collected from the firm and transfers this amount lump sum to the consumer. Show that the government can set t so that the competitive equilibrium is Pareto optimal, and determine the optimal tax rate. Explain your result.
- (d) Now, suppose that there is a market in pollution rights. That is, the consumer sells the firm pollution rights in that, for the right to produce x units of pollution, the firm pays the consumer xp , where p is the competitively-determined price of pollution rights. Show that the competitive equilibrium is Pareto optimal, and explain why.
3. ($33\frac{1}{3}$ points) Consider a representative agent economy where the representative consumer maximizes

$$\sum_{t=0}^{\infty} \beta^t \log [\min(c_t, \alpha l_t)],$$

where $0 < \beta < 1$, c_t is consumption, l_t is leisure, and $\alpha > 0$. The production technology is given by

$$y_t = n_t,$$

where y_t is output and n_t is labor input. The government has a technology which allows it to convert consumption goods one-for-one into public goods, g_t . The government budget constraint is

$$g_t + (1 + r_t)b_t = \tau_t + b_{t+1},$$

where b_{t+1} is the quantity of government bonds issued by the government in period t , with each of these bonds representing a promise to pay $1 + r_{t+1}$ units of the consumption good in period $t + 1$. Assume $b_0 = 0$. The representative consumer pays a lump-sum tax of τ_t in period t . Let w_t denote the wage rate in period t . In even periods, $t = 0, 2, 4, \dots$, the government sets $g_t = g^*$, while in odd periods, $t = 1, 3, 5, \dots$, the government sets $g_t = g^{**}$, with $g^* > g^{**}$.

- (a) Show that output, consumption, employment, and the real interest rate each follow a two-cycle in a competitive equilibrium. For example, output follows a sequence $\{y^*, y^{**}, y^*, y^{**}, \dots\}$.
- (b) Determine whether the real interest rate is higher in even periods or in odd periods, and explain your results (recall that the period t real interest rate is r_{t+1}).