6E:204 Macroeconomics
Test 2

Steve Williamson

October 31, 2000

Instructions: Read the questions carefully and make sure to show all your work. Good luck!

1. ($33\frac{1}{3}$ points) Consider an overlapping generations model where time is indexed by $t = 0, 1, 2, ...$, and there are L_t two-period-lived consumers born in each period t. The population evolves according to

$$L_t = (1 + n)^t L_0,$$

where L_0 is given and $n > 0$. A consumer born in period t has preferences given by

$$u(c_t^y, c_{t+1}^o) = \min(c_t^y, c_{t+1}^o),$$

where c_t^y is consumption when young, and c_{t+1}^o is consumption when old. At $t = 0$ there are some one-period-lived agents alive who are collectively endowed with K_0 units of capital. Each young agent has an endowment of one unit of time. The production technology is given by

$$Y_t = K_t^\alpha L_t^{1-\alpha},$$

where K_t is the capital input, L_t is the labor input, and $0 < \alpha < 1$. Consumption goods in period t can be transformed one-for-one into capital, but the capital will not be productive until period $t + 1$, and there is no depreciation.

   (a) Determine the optimal steady state capital/labor ratio and consumption allocation.

   (b) Determine the capital/labor ratio and consumption of the young and old in a competitive equilibrium steady state. How does this differ from the optimal steady state? Explain why you get this result.

   (c) Now suppose that there is a pay-as-you-go social security program under which, at each date, the young are taxed in equal lump sum amounts, with transfers made in equal lump sum amounts to the old. Determine whether or not this arrangement can achieve the optimum in a steady state. If it can, determine the optimal social security scheme.

2. ($33\frac{1}{3}$ points) Suppose an economy where the representative consumer maximizes

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where c_t is consumption, $0 < \beta < 1$, and $u(\cdot)$ is a strictly increasing and strictly concave utility function. The production technology is given by

$$y_t = \alpha k_t,$$

where y_t is output, k_t is the capital stock, and $\alpha > 0$. The capital stock depreciates by 100% each period.

   (a) Determine conditions under which consumption and output will exhibit unbounded growth, and explain your results.

   (b) Assume that $u(c_t) = \ln c_t$, and solve for consumption, output, the capital stock, and investment, in each period t, in a competitive equilibrium. Show that all quantities grow at the same constant rate, and explain the determinants of this growth rate.

3. ($33\frac{1}{3}$ points) Consider an economy where the representative household has preferences given by

$$\sum_{t=0}^{\infty} \beta^t N_t \frac{c_t^{\gamma}}{\gamma},$$

where $0 < \beta < 1$, c_t is per-capita consumption, N_t is the population, and $\gamma < 1$. The production technology is given by

$$Y_t = \min(K_t, A_t N_t)$$

where Y_t is output K_t is the capital input, and A_t denotes labor-augmenting technological progress. Assume that $N_t = (1+n)^t N_0$ and $A_t = (1+a)^t A_0$ where $n > 0$, $a > 0$, and N_0 and A_0 are positive and given. Capital depreciates by 100% each period.

   (a) Show that this economy cannot sustain a growth path where $K_t = A_t N_t$.

   (b) Show that per capita consumption decreases at a constant rate in the steady state, and determine this rate of decrease. Comment on the features of your solution, and explain why this economy does not grow.