

06E:204 Macroeconomics  
Assignment 1

Steve Williamson  
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1. Consider the following representative agent model. The representative consumer has preferences given by

$$u(c, l) = c + \beta l$$

where  $c$  is consumption,  $l$  is leisure, and  $\beta > 0$ . The consumer has an endowment of one unit of time and  $k_0$  units of capital. The representative firm has a technology for producing consumption goods, given by

$$y = zk^\alpha n^{1-\alpha}$$

where  $y$  is output,  $z$  is total factor productivity,  $k$  is the capital input,  $n$  is the labor input, and  $0 < \alpha < 1$ . The market real wage is  $w$ , and  $r$  denotes the capital rental rate.

- (a) Solve for all prices and quantities in a competitive equilibrium (there are two cases to consider).
  - (b) Determine the effects of a change in  $z$  on consumption, output, employment, the real wage, and the capital rental rate, and explain your results.
2. Consider an economy with a continuum of consumers, and normalize the total mass of consumers to one. Each consumer has preferences given by

$$U(c, l, \bar{c}) = u(c, l) + v(\bar{c}),$$

where  $c$  and  $l$  are the individual's consumption and leisure, respectively, and  $\bar{c}$  is average consumption across the population (note that, because any individual is very small relative to the population, each consumer will treat  $\bar{c}$  as given). Assume that  $u(\cdot, \cdot)$  has standard properties, and that  $v(\cdot)$  is strictly increasing, strictly concave, and twice differentiable. There is an externality in consumption

in that any individual is better off when others consume more. The production technology is given by

$$y = n,$$

where  $y$  is output and  $n$  is the labor input.

- (a) Determine the Pareto optimum (confine attention to allocations where all consumers consume the same quantities).
- (b) Determine the competitive equilibrium, and show that it is not Pareto optimal.
- (c) Now, suppose that the government subsidizes each individual's consumption. That is, for each unit he or she consumes, a consumer receives  $s$  units of consumption from the government. The government finances subsidies to consumers by imposing a lump-sum tax  $\tau$  on each consumer. Show that, if the government sets the subsidy appropriately, then the competitive equilibrium is Pareto optimal. Determine the optimal subsidy, and explain your results.