06E:204 Macroeconomics
Assignment 3

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1. Consider the following representative agent model. There is a representative consumer, with preferences given by

\[ X^* = \beta^t [u(c_t) + v(l_t) + \phi(g_t)], \]

where \( \beta \) is the discount factor, with \( 0 < \beta < 1 \), \( c_t \) is consumption, \( l_t \) is leisure, \( g_t \) is the quantity of public goods, and \( u(\cdot), v(\cdot), \) and \( \phi(\cdot) \) are each strictly increasing, strictly concave, twice continuously differentiable, and have the property that \( u'(0) = v'(0) = \phi'(0) = \infty \). The government’s budget constraint in period \( t \) is

\[ g_t + (1 + r_t)b_t = \tau_t + b_{t+1}, \]

for \( t = 0, 1, 2, \ldots \), where \( r_t \) is the real interest rate, \( b_t \) is the quantity of one-period bonds issued in period \( t - 1 \) by the government, and \( \tau_t \) is a lump sum tax paid by the representative consumer. Assume that \( b_0 = 0 \). The technology is given by \( y_t = z_t n_t \) where \( y_t \) is output, \( z_t \) is labor productivity, and \( n_t \) is labor input. The sequence \( \{z_t\}_{t=0}^\infty \) is exogenous. Suppose that the government chooses the path for government spending \( \{g_t\}_{t=0}^\infty \) so as to maximize the welfare of the representative consumer, and suppose \( \{z_t\}_{t=0}^\infty = \{z^*, z^{**}, z^*, z^{**}, \ldots\} \), where \( z^* > z^{**} \). That is, labor productivity is high in even periods and low in odd periods (it follows a two-cycle).

(a) Show that consumption, leisure, employment, government spending, and the real interest rate also follow two cycles.

(b) Determine whether consumption, leisure, employment, government spending, and the real interest rate, respectively, are higher in even or odd periods. Explain your results.
(c) Suppose two alternative paths for taxes: (i) $\tau_t = g_t$ for $t = 0, 1, 2, \ldots$; (ii) $\tau_t = \tau^*$ for all $t$, where $\tau^* > 0$ is a constant. Does it matter for the competitive equilibrium whether the government chooses (i) or (ii)? What is the path followed by the government deficit and the government debt given policies (i) and (ii)?