

06E:204 Macroeconomics
Assignment 3

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September 18, 2001
Due: September 25, 2001

1. Consider the following representative agent model. There is a representative consumer, with preferences given by

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) + v(l_t) + \phi(g_t)],$$

where β is the discount factor, with $0 < \beta < 1$, c_t is consumption, l_t is leisure, g_t is the quantity of public goods, and $u(\cdot)$, $v(\cdot)$, and $\phi(\cdot)$ are each strictly increasing, strictly concave, twice continuously differentiable, and have the property that $u'(0) = v'(0) = \phi'(0) = \infty$. The government's budget constraint in period t is

$$g_t + (1 + r_t)b_t = \tau_t + b_{t+1},$$

for $t = 0, 1, 2, \dots$, where r_t is the real interest rate, b_t is the quantity of one-period bonds issued in period $t - 1$ by the government, and τ_t is a lump sum tax paid by the representative consumer. Assume that $b_0 = 0$. The technology is given by $y_t = z_t n_t$ where y_t is output, z_t is labor productivity, and n_t is labor input. The sequence $\{z_t\}_{t=0}^{\infty}$ is exogenous. Suppose that the government chooses the path for government spending $\{g_t\}_{t=0}^{\infty}$ so as to maximize the welfare of the representative consumer, and suppose $\{z_t\}_{t=0}^{\infty} = \{z^*, z^{**}, z^*, z^{**}, \dots\}$, where $z^* > z^{**}$. That is, labor productivity is high in even periods and low in odd periods (it follows a two-cycle).

- (a) Show that consumption, leisure, employment, government spending, and the real interest rate also follow two cycles.
- (b) Determine whether consumption, leisure, employment, government spending, and the real interest rate, respectively, are higher in even or odd periods. Explain your results.

- (c) Suppose two alternative paths for taxes: (i) $\tau_t = g_t$ for $t = 0, 1, 2, \dots$;
(ii) $\tau_t = \tau^*$ for all t , where $\tau^* > 0$ is a constant. Does it matter for the competitive equilibrium whether the government chooses (i) or (ii)? What is the path followed by the government deficit and the government debt given policies (i) and (ii)?