6E:204 Macroeconomics
Assignment 4

Steve Williamson
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1. Consider the following overlapping generations growth model. Time is indexed by $t = 0, 1, 2, ...$, and in period t there are L_t two-period-lived consumers born, where $L_t = (1+n)^t L_0$, with L_0 given and $n > 0$. At $t = 0$ there are some one-period-lived old agents who are collectively endowed with K_0 units of capital, and who maximize consumption at $t = 0$. A consumer born in period t has preferences given by
$$u(c_t^y, c_{t+1}^o) = \min(c_t^y, \beta c_{t+1}^o)$$
where c_t^y (c_t^o) denotes consumption of a young (old) consumer in period t, and $\beta > 0$. Each consumer has one unit of time available when young. The production technology is given by
$$Y_t = K_t^\alpha L_t^{1-\alpha},$$
where Y_t is output, K_t is the capital input, and L_t is the labor input. One unit of the consumption good can be converted into one unit of capital, and vice-versa. In period t, the government issues B_{t+1} one-period bonds, each of which is a claim to $1 + r_{t+1}$ units of the consumption goods in period $t+1$. There is a lump-sum tax of τ_t paid by each young consumer in period t. The government holds constant the per-worker quantity of government bonds, that is $B_{t+1} = bL_t$.

   (a) Determine the capital/labor ratio, consumption of the young, and consumption of the old, in a socially optimal steady state.

   (b) Suppose that $b = 0$. Determine the capital/labor ratio, consumption of the young, and consumption of the old in the competitive equilibrium steady state. Is the competitive equilibrium steady state socially optimal? Why or why not?

   (c) Now, suppose that $b \neq 0$. Show that the government can set b so as to achieve the socially optimal steady state in a competitive equilibrium, and let $\hat{b}$ denote the optimal b. Determine $\hat{b}$. Is $\hat{b}$ positive or negative? Explain your results.

2. Consider the following overlapping generations growth model. Time is indexed by $t = 0, 1, 2, ...$, and in period t there are L_t two-period-lived consumers born, where $L_t = (1 + n)^t L_0$, with L_0 given and $n > 0$. In periods $t = 0, 1, 2, ...$, each young consumer is endowed with y units of the consumption good. Each old consumer (including the initial old in period 0) is endowed with nothing. There is a storage technology which permits one unit of period t consumption goods to be converted to $1 + r$ units of period $t + 1$ consumption goods, for $t = 0, 1, 2, ...$. In period t, the government collects a lump-sum tax of τ_t^y from each young consumer, and a lump-sum tax of τ_t^o from each old consumer. A consumer born in period t has preferences given by

$$u(c_t^y, c_{t+1}^o) = 2(c_t^y)^{\frac{1}{2}} + 2(c_{t+1}^o)^{\frac{1}{2}}.$$

(a) Write down the government's budget constraint.

(b) Suppose a pay-as-you-go social security scheme, where each period the government taxes the young so as to make transfers to the old. That is, the government sets $\tau_t^y = \tau$ for $t = 0, 1, 2, ...$. Determine the effects of an increase in τ on the savings of each young consumer, and on the welfare of each generation. Also determine the Pareto optimal level of τ and explain your results.

(c) Suppose a fully-funded social security system where the government taxes the young in period t, puts the proceeds of the tax into storage, and then makes transfers to the old in period $t + 1$ with the proceeds from period t storage. In this case the government sets $\tau_t^y = \tau$ for $t = 0, 1, 2, ...$. Determine the effects of an increase in τ on the savings of each young consumer, the consumption of the young and the old in each generation, and on the welfare of each generation. As in part (b), determine the Pareto optimal level of τ and explain your results.