6E:204 Macroeconomics
Assignment 5

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1. Consider a representative agent model where the representative consumer has preferences given by
$$\sum_{t=0}^{\infty} \beta^t \log c_t,$$
where $0 < \beta < 1$ and c_t is consumption in period t. The production technology is given by
$$y_t = \alpha_t k_t,$$
where $\alpha_t = \alpha^*$ for $t = 0, 2, 4, ...$, and $\alpha_t = \alpha^{**}$ for $t = 1, 3, 5, ...$. Assume that $\alpha^*\beta > 1$ and $\alpha^{**}\beta < 1$. Also assume 100% depreciation.

   (a) Solve for competitive equilibrium quantities by using guess-and-verify methods (note that in general the value function will be different in even and odd periods). Show that the capital stock, output, and consumption increase in even periods and decrease in odd periods.

   (b) Show that trend consumption increases (that is, consumption increases from period t to period $t + 2$ for all t) if $\alpha^*\alpha^{**}\beta^2 > 1$ and decreases if $\alpha^*\alpha^{**}\beta^2 < 1$.

   (c) Explain your results.

2. A consumer has preferences given by
$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$
where $0 < \beta < 1$, c_t is consumption, and $u(\cdot)$ is a strictly increasing, strictly concave function with $u'(0) = \infty$. The consumer has A_0 assets at the beginning of period 0, and faces a market real interest rate of r in each period $t = 0, 1, 2, ...$

. The consumer earns no income during his or her lifetime, so his or her period t budget constraint is given by

$$A_{t+1} = (1+r)(A_t - c_t),$$

where A_t is the quantity of assets at the beginning of period t.

(a) Show that, if $\beta(1+r) > 1$, then consumption will increase over time for the consumer, if $\beta(1+r) < 1$, then consumption decreases over time, and if $\beta(1+r) = 1$ then consumption will be constant over time. Explain these results.