

6E:204 Macroeconomics
Assignment 6

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1. The dynastic household has preferences given by

$$\sum_{t=0}^{\infty} \beta^t N_t \frac{c_t^\gamma}{\gamma},$$

where $0 < \beta < 1$, N_t is population, c_t is per capita consumption, and $\gamma < 1$. Population grows according to

$$N_t = (1 + n)^t N_0,$$

where N_0 is given and $n > -1$. The production technology is

$$Y_t = K_t^\alpha L_t^{1-\alpha},$$

where Y_t is output, K_t is the capital input, L_t is the effective labor input, and $0 < \alpha < 1$. Each consumer divides her time between human capital accumulation and work each period, and has a total endowment of one unit of time per period. Letting u_t denote time allocated by a consumer to human capital accumulation at the beginning of period t , the quantity of human capital produced by that consumer during period t is

$$h_t = u_t^\delta,$$

where $\delta > 0$. The effective labor input is then $L_t = N_t h_t (1 - u_t)$. Human capital depreciates by 100% at the end of the period. One unit of physical capital can be produced using one unit of output in period t , and this capital becomes productive in period $t + 1$, and then depreciates by 100%.

- (a) Solve for steady state per capita output, per capita consumption, per capita capital, and time allocated to human capital accumulation. Also solve for the steady state savings rate. How do your results depend on δ ? Explain.

- (b) In cross-country data, the savings rate and per capita output are positively correlated, and per capita output and average years of schooling of the population are also positively correlated. Are the predictions of this model consistent in these respects with the data? Explain.

2. Consider a model where the dynastic household has preferences given by

$$\sum_{t=0}^{\infty} \beta^t N_t \frac{c_t^\gamma}{\gamma},$$

where $0 < \beta < 1$, N_t is population, c_t is per capita consumption, and $\gamma < 1$. We have $N_t = (1 + n)^t N_0$ with $n > -1$ and N_0 given. The production technology is given by

$$Y_t = \alpha h_t u_t N_t,$$

where Y_t is aggregate production of the perishable consumption good, $\alpha > 0$, h_t is the per capita quantity of human capital, and u_t is the fraction of time spent by each consumer in the production of consumption goods. Each consumer has one unit of time available in each period. Human capital is produced according to

$$h_{t+1} = \delta h_t (1 - u_t),$$

where $\delta > 0$. Assume that, each period, the government purchases a fraction θ of the total quantity of consumption goods produced, and finances these purchases with lump-sum taxation.

- (a) Determine the competitive equilibrium growth path for per-capita consumption, per-capita output, human capital, and the fraction of time spent producing consumption goods and human capital.
- (b) How are your results in part (a) affected by θ ? Explain.