

6E:204 Macroeconomics  
Assignment 7

Steve Williamson  
November 13, 2001  
Due: November 20, 2001

1. Consider a representative agent model where the representative consumer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [\ln c_t + \ln l_t]$$

The consumer has one unit endowment of time to allocate between consumption and leisure each period. The production technology is given by

$$y_t = z_t k_t^\alpha n_t^{1-\alpha},$$

where  $y_t$  is output,  $z_t$  is a technology shock,  $k_t$  is the capital input, and  $n_t$  is the labor input, with  $0 < \alpha < 1$ . The capital stock depreciates by 100% each period. In period  $t$ , it requires one unit of consumption goods to produce one unit of capital, and this capital becomes productive in period  $t + 1$ . Assume that

$$z_{t+1} = z_t^\rho \epsilon_{t+1},$$

where  $\ln \epsilon_t$  is an i.i.d. random variable with a mean of zero and  $0 < \rho < 1$ .

- (a) Solve for a competitive equilibrium.
  - (b) How does employment vary with the technology shock  $z_t$ ? Is this model capable of explaining observed fluctuations in employment? Explain.
  - (c) How does persistence in the technology shock ( $\rho > 0$ ) affect consumption, investment, and output over time? Which of these properties do you think are special to this example? Explain.
2. Suppose a consumer with preferences given by

$$- \sum_{t=0}^{\infty} \beta^t e^{-\alpha c_t},$$

where  $0 < \beta < 1$ ,  $c_t$  is consumption, and  $\alpha > 0$ . The consumer has initial assets  $A_0$  and can borrow and lend at a real interest  $r$  in each period. The consumer's income in period  $t$  is  $w_t$ , for  $t = 0, 1, 2, \dots$ .

- (a) Show that the change in consumption from periods  $t$  to  $t + 1$  depends only on  $\alpha$ ,  $\beta$ , and  $r$ , and derive this relationship. In what sense is consumption smooth relative to income?
- (b) How is the change in consumption affected by changes in each of  $\alpha$ ,  $\beta$ , and  $r$ ? Explain your results.