1. Consider a representative agent model where the representative consumer has preferences given by

$$E_0 \int_{t=0}^{\infty} \beta^t [\ln c_t + \ln l_t]$$

The consumer has one unit endowment of time to allocate between consumption and leisure each period. The production technology is given by

$$y_t = z_t k_t^\alpha n_t^{1-\alpha},$$

where $y_t$ is output, $z_t$ is a technology shock, $k_t$ is the capital input, and $n_t$ is the labor input, with $0 < \alpha < 1$. The capital stock depreciates by 100% each period. In period $t$, it requires one unit of consumption goods to produce one unit of capital, and this capital becomes productive in period $t+1$. Assume that

$$z_{t+1} = z_t \epsilon_{t+1},$$

where $\ln \epsilon_t$ is an i.i.d. random variable with a mean of zero and $0 < \rho < 1$.

(a) Solve for a competitive equilibrium.
(b) How does employment vary with the technology shock $z_t$? Is this model capable of explaining observed fluctuations in employment? Explain.
(c) How does persistence in the technology shock ($\rho > 0$) affect consumption, investment, and output over time? Which of these properties do you think are special to this example? Explain.

2. Suppose a consumer with preferences given by

$$\int_{t=0}^{\infty} \beta^t e^{-\alpha c_t},$$

where $0 < \beta < 1$, $c_t$ is consumption, and $\alpha > 0$. The consumer has initial assets $A_0$ and can borrow and lend at a real interest rate $r$ in each period. The consumer’s income in period $t$ is $w_t$, for $t = 0, 1, 2, \ldots$.
(a) Show that the change in consumption from periods $t$ to $t+1$ depends only on $\alpha$, $\beta$, and $r$, and derive this relationship. In what sense is consumption smooth relative to income?

(b) How is the change in consumption affected by changes in each of $\alpha$, $\beta$, and $r$? Explain your results.