1. Consider a representative agent model where the representative consumer has preferences given by
\[
E_0^\infty \beta^t \ln c_t
\]
Output, \(y_t\), is produced on one productive units, and there is a supply of one share in this productive unit. Assume that \(y_t\) is i.i.d., and that \(\Pr[y_t = y^*] = \pi\) and \(\Pr[y_t = y^{**}] = 1 - \pi\), with \(0 < \pi < 1\) and \(y^* > y^{**}\). Let \(p_t\) denote the price of a share in aggregate output in period \(t\). There also exist insurance claims, which are traded on competitive markets in each period. One insurance claim can be purchased at a price \(q_t\) in period \(t\), and it pays off one unit of the consumption good in period \(t + 1\) if \(y_{t+1} = y^{**}\) and pays off nothing otherwise. Determine \(p_t\) and \(q_t\) in each state of the world, and explain why asset prices move across states in the way that they do.

2. Consider a representative agent economy where the representative agent has preferences given by
\[
\int_{t=0}^\infty \beta^t u(c_t)
\]
where \(0 < \beta < 1\), \(c_t\) is consumption, and \(u(\cdot)\) is an increasing and strictly concave function. Aggregate output, \(y_t\), is the exogenous output of a single firm. There is a single divisible share in this firm which is traded on a competitive market in each period. We will let asset 0 denote this share in aggregate output. Assume that \(y_t\) is an i.i.d. random variable. There are also \(n\) other assets which are in zero net supply. These assets are indexed by \(i = 1, 2, \ldots, n\), and the \(i^{th}\) asset is an \(i\)-period real bond. That is, in period \(t\), asset \(i\) is a claim to one unit of consumption in period \(t + i\), for \(i = 1, 2, \ldots, n\). Each asset is traded competitively until it matures, i.e. until the payoff is received. Let \(p_i^t\) denote the price of asset \(i\) in period \(t\).
(a) Determine the price of asset 1 as a function of current output, $y$, and let $p^1(y)$ denote this price.

(b) Now, given $p^1(y)$, determine $p^2(y)$ the price of asset 2, and derive a general formula for $p^i(y)$, the price of asset $i$.

(c) The yield to maturity on asset $i$ is given by

$$r^i(y) = \frac{\bar{A}}{p^i(y)} - 1,$$

and the yield curve is a plot of the yield to maturity as a function of time to maturity, i.e. a plot of $r^i(y)$ as a function of $i$. The expectations theory of the term structure of interest rates predicts that the yield curve is upward-sloping when short-term interest rates are expected to rise, and that it is downward-sloping when short-term interest rates are expected to fall.

i. Show how the level of output determines whether the yield curve is upward-sloping or downward-sloping, and explain your results.

ii. Is the model consistent with the expectations theory? Explain why or why not.