

6E:204 Macroeconomics
Assignment 8

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November 20, 2001
Due: November 27, 2001

1. Consider a representative agent model where the representative consumer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t$$

Output, y_t , is produced on one productive units, and there is a supply of one share in this productive unit. Assume that y_t is i.i.d., and that $\Pr[y_t = y^*] = \pi$ and $\Pr[y_t = y^{**}] = 1 - \pi$, with $0 < \pi < 1$ and $y^* > y^{**}$. Let p_t denote the price of a share in aggregate output in period t . There also exist insurance claims, which are traded on competitive markets in each period. One insurance claim can be purchased at a price q_t in period t , and it pays off one unit of the consumption good in period $t + 1$ if $y_{t+1} = y^{**}$ and pays off nothing otherwise. Determine p_t and q_t in each state of the world, and explain why asset prices move across states in the way that they do.

2. Consider a representative agent economy where the representative agent has preferences given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $0 < \beta < 1$, c_t is consumption, and $u(\cdot)$ is an increasing and strictly concave function. Aggregate output, y_t , is the exogenous output of a single firm. There is a single divisible share in this firm which is traded on a competitive market in each period. We will let asset 0 denote this share in aggregate output. Assume that y_t is an i.i.d. random variable. There are also n other assets which are in zero net supply. These assets are indexed by $i = 1, 2, \dots, n$, and the i^{th} asset is an i -period real bond. That is, in period t , asset i is a claim to one unit of consumption in period $t + i$, for $i = 1, 2, \dots, n$. Each asset is traded competitively until it matures, i.e. until the payoff is received. Let p_t^i denote the price of asset i in period t .

- (a) Determine the price of asset 1 as a function of current output, y , and let $p^1(y)$ denote this price.
- (b) Now, given $p^1(y)$, determine $p^2(y)$ the price of asset 2, and derive a general formula for $p^i(y)$, the price of asset i .
- (c) The yield to maturity on asset i is given by

$$r^i(y) = \frac{\bar{A}^i}{p^i(y)} - 1,$$

and the **yield curve** is a plot of the yield to maturity as a function of time to maturity, i.e. a plot of $r^i(y)$ as a function of i . The **expectations theory of the term structure of interest rates** predicts that the yield curve is upward-sloping when short-term interest rates are expected to rise, and that it is downward-sloping when short-term interest rates are expected to fall.

- i. Show how the level of output determines whether the yield curve is upward-sloping or downward-sloping, and explain your results.
- ii. Is the model consistent with the expectations theory? Explain why or why not.