

**6E:204 Macroeconomics**  
**Final Exam**

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**Instructions:** Read the questions carefully and make sure to show all your work.  
Good luck!

1. (35 points) Consider the following representative agent model. The representative agent has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where  $0 < \beta < 1$ ,  $c_t$  is consumption, and  $u(\cdot)$  is increasing, strictly concave, and twice differentiable. Output is produced on  $n$  productive units, indexed by  $i = 1, 2, \dots, n$ . For productive units  $i = 1, 2, \dots, n - 1$ , output in period  $t$  is given by  $y_{it} = \theta_{it}$ , and output for the  $n^{\text{th}}$  productive unit is  $y_{nt} = y - \sum_{i=1}^{n-1} \theta_{it}$ . Here,  $(\theta_{1t}, \theta_{2t}, \dots, \theta_{n-1,t})$  is a multivariate random variable, and  $y$  is a constant. There is one share outstanding in each productive unit, and these shares are traded on competitive markets in each period. Let  $p_{it}$  denote the price of a share in the  $i^{\text{th}}$  productive unit in period  $t$ .

- (a) Show that the expected rate of return for each asset is a constant, and determine what this constant is.
  - (b) Determine the risk premium for each asset.
  - (c) Explain your results in parts (a) and (b).
2. (35 points) Consider the following search model. There is a continuum of agents, each having preferences given by

$$E_0 \sum_{t=0}^{\infty} \left( \frac{1}{1+r} \right)^t [u(c_t) - a_t],$$

where  $r > 0$ ,  $u(\cdot)$  is an increasing function with  $u(0) = 0$ ,  $c_t$  is consumption, and  $a_t$  denotes costs of production. Agents can be either unemployed or employed, where  $\gamma_t$  denotes the fraction of the population that is employed. Each

unemployed agent receives a production opportunity during the current period. With probability  $\rho$ , an unemployed agent receives a production opportunity implying that it will cost  $\alpha_1$  to produce  $y$  units of output, and with probability  $1 - \rho$  the unemployed agent receives a production opportunity where the cost is  $\alpha_2$  to produce  $y$  units of output. Assume that  $0 < \alpha_1 < \alpha_2 < u(y)$  and  $0 < \rho < 1$ . When an unemployed agent produces in period  $t$ , he or she becomes employed in period  $t + 1$ , and begins searching for a trading partner, since he or she cannot consume his or her own output. An employed agent meets a trading partner during the current period with probability  $b\gamma_t$ , where  $0 < b < 1$ . When two employed agents meet in period  $t$ , they trade output, consume, and then become unemployed in period  $t + 1$ .

- (a) Write down Bellman equations that determine the values of being employed and unemployed in a steady state.
  - (b) Show that there exist five steady state equilibria, and determine these equilibria.
  - (c) Explain why there are multiple equilibria in this model, and explain what economic phenomena the model might help us understand.
3. (35 points) Consider the following cash-in-advance model. The representative consumer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t [\ln(c_t) - \alpha n_t^s],$$

where  $0 < \beta < 1$ ,  $c_t$  is consumption,  $\alpha > 0$ , and  $n_t^s$  is labor supply. The production technology is given by

$$y_t = \gamma_t n_t^d,$$

where  $n_t^d$  is the labor input,  $y_t$  is output of the perishable consumption good, and

$$\gamma_t = \phi^t \gamma_0,$$

where  $\gamma_0 > 0$  and  $\phi > 0$ . The money supply grows according to

$$\bar{M}_t = \theta^t \bar{M}_0,$$

where  $\bar{M}_0$  is the money supply that the representative consumer is endowed with at the beginning of period 0, and  $\theta \geq \beta$ . At the beginning of period  $t$ , the consumer has  $M_t$  units of money,  $B_t$  one-period nominal bonds, and  $z_t$  one-period real bonds. Each nominal bond sells in period  $t$  for  $S_t$  units of money,

and pays off one unit of money in period  $t + 1$ , while each real bond sells for  $q_t$  units of consumption goods in period  $t$  and pays off one unit of consumption goods in period  $t + 1$ . Let  $P_t$  denote the price level. At the beginning of the period, the consumer receives a money transfer of  $P_t\tau_t$  from the government, and then trades on the asset market. Then, the consumer supplies labor to the representative firm, and purchases goods with money on the goods market. When the goods market closes, the consumer receives wage payments in cash from the firm.

- (a) Determine employment, consumption, output, the real wage, the price level, the inflation rate, and real and nominal interest rates, in a competitive equilibrium.
- (b) How do  $\theta$  and  $\bar{M}_0$  affect the competitive equilibrium you determined in part (a)?
- (c) Suppose that the government can set  $\bar{M}_t$  optimally each period. How should it do this, and what is the resulting competitive equilibrium?
- (d) Explain your results in parts (a), (b) and (c).