

6E:204 Macroeconomics
Test 1

STEVE WILLIAMSON

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Instructions: Read the questions carefully and make sure to show all your work.
Good luck!

1. (35 points) Consider a one-period economy where the representative consumer has preferences given by the utility function $u(c, l)$, where c is consumption and l is leisure. The consumer has an endowment of 1 unit of time which can be allocated between work and leisure. The representative firm produces consumption goods according to $y = zn$, where y is output and n is labor input. The government purchases an exogenous quantity of the consumption good, g , and finances this expenditure by imposing a proportional tax t on the representative firm's output. Then, the firm's after-tax profits are $z(1-t)n - wn$, where w is the market real wage rate.
 - (a) Derive a set of equations that solve for the real wage, consumption, output, leisure, and employment, in a competitive equilibrium.
 - (b) Suppose alternatively that government purchases are financed with a lump sum tax on the representative consumer. Show that the representative consumer is better off in this case than when there is a proportional tax on the firm's output. Explain your results.
 - (c) Compare the competitive equilibrium with a lump sum tax with the competitive equilibrium when there is a proportional tax on the firm's output. In which case is employment higher? What about consumption, output, and the real wage? Explain your results.
2. (35 points) Suppose a one-period economy where the representative consumer has preferences given by

$$u(c, l) = \ln c + \beta \ln l$$

where c is consumption and l is leisure, with $\beta > 0$. The consumer is endowed with one unit of time and k_0 units of capital. The representative firm has a production technology given by

$$y = zn + k,$$

where $z > 0$, y is output of consumption goods, n is labor input, and k is the capital input. Let w denote the real wage and r the rental rate on capital.

- (a) Determine output, consumption, the quantity of leisure, employment, the real wage, and the rental rate on capital in a competitive equilibrium. Make sure to consider all the relevant cases (hint: it may help to draw a picture).
 - (b) Determine the effects of a change in z on consumption, leisure, employment, the real wage, and the rental rate on capital, and explain your results.
3. (35 points) Consider a representative agent economy where the representative consumer maximizes

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

where $0 < \beta < 1$, c_t is consumption and l_t is leisure. Assume that $u_{12} > 0$. The production technology is given by

$$y_t = z_t n_t,$$

where y_t is output and n_t is labor input. The government has a technology which allows it to convert consumption goods one-for-one into public goods, g_t . The government budget constraint is

$$g_t + (1 + r_t)b_t = \tau_t + b_{t+1},$$

where b_{t+1} is the quantity of government bonds issued by the government in period t , with each of these bonds representing a promise to pay $1 + r_{t+1}$ units of the consumption good in period $t + 1$. Assume $b_0 = 0$. The representative consumer pays a lump-sum tax of τ_t in period t . Let w_t denote the wage rate in period t . The government sets government spending in each period so that aggregate consumption is a constant, c^* , in each period. Suppose that $z_t = z^*$ for $t = 0, 1, 2, \dots, T - 1$, and $z_t = z^{**} > z^*$ for $t = T, T + 1, T + 2, \dots$.

- (a) Determine the path followed by consumption, output, employment, the real interest rate, and the real wage for $t = 0, 1, 2, \dots$.
- (b) Explain your results.