6E:204 Macroeconomics
Test 1

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Instructions: Read the questions carefully and make sure to show all your work. Good luck!

1. (35 points) Consider a one-period economy where the representative consumer has preferences given by the utility function \( u(c, l) \), where \( c \) is consumption and \( l \) is leisure. The consumer has an endowment of 1 unit of time which can be allocated between work and leisure. The representative firm produces consumption goods according to \( y = zn \), where \( y \) is output and \( n \) is labor input. The government purchases an exogenous quantity of the consumption good, \( g \), and finances this expenditure by imposing a proportional tax \( t \) on the representative firm’s output. Then, the firm’s after-tax profits are \( z(1-t)n-wn \), where \( w \) is the market real wage rate.

(a) Derive a set of equations that solve for the real wage, consumption, output, leisure, and employment, in a competitive equilibrium.

(b) Suppose alternatively that government purchases are financed with a lump sum tax on the representative consumer. Show that the representative consumer is better off in this case than when there is a proportional tax on the firm’s output. Explain your results.

(c) Compare the competitive equilibrium with a lump sum tax with the competitive equilibrium when there is a proportional tax on the firm’s output. In which case is employment higher? What about consumption, output, and the real wage? Explain your results.

2. (35 points) Suppose a one-period economy where the representative consumer has preferences given by \( u(c, l) = \ln c + \beta \ln l \) where \( c \) is consumption and \( l \) is leisure, with \( \beta > 0 \). The consumer is endowed with one unit of time and \( k_0 \) units of capital. The representative firm has a production technology given by \( y = zn + k \),
where \( z > 0 \), \( y \) is output of consumption goods, \( n \) is labor input, and \( k \) is the capital input. Let \( w \) denote the real wage and \( r \) the rental rate on capital.

(a) Determine output, consumption, the quantity of leisure, employment, the real wage, and the rental rate on capital in a competitive equilibrium. Make sure to consider all the relevant cases (hint: it may help to draw a picture).

(b) Determine the effects of a change in \( z \) on consumption, leisure, employment, the real wage, and the rental rate on capital, and explain your results.

3. (35 points) Consider a representative agent economy where the representative consumer maximizes

\[
\sum_{t=0}^{\infty} \beta^t u(c_t, l_t),
\]

where \( 0 < \beta < 1 \), \( c_t \) is consumption and \( l_t \) is leisure. Assume that \( u_{12} > 0 \). The production technology is given by

\[
y_t = z_t n_t,
\]

where \( y_t \) is output and \( n_t \) is labor input. The government has a technology which allows it to convert consumption goods one-for-one into public goods, \( g_t \). The government budget constraint is

\[
g_t + (1 + r_t) b_t = \tau_t + b_{t+1},
\]

where \( b_{t+1} \) is the quantity of government bonds issued by the government in period \( t \), with each of these bonds representing a promise to pay \( 1 + r_{t+1} \) units of the consumption good in period \( t + 1 \). Assume \( b_0 = 0 \). The representative consumer pays a lump-sum tax of \( \tau_t \) in period \( t \). Let \( w_t \) denote the wage rate in period \( t \). The government sets government spending in each period so that aggregate consumption is a constant, \( c^* \), in each period. Suppose that \( z_t = z^* \) for \( t = 0, 1, 2, \ldots, T - 1 \), and \( z_t = z^{**} > z^* \) for \( t = T, T + 1, T + 2, \ldots \).

(a) Determine the path followed by consumption, output, employment, the real interest rate, and the real wage for \( t = 0, 1, 2, \ldots \).

(b) Explain your results.