

6E:204 Macroeconomics

Test 2

STEVE WILLIAMSON

November 6, 2001

Instructions: Read the questions carefully and make sure to show all your work.
Good luck!

1. (35 points) Consider the following overlapping generations growth model. Time is indexed by $t = 0, 1, 2, \dots$, and in period t there are L_t two-period-lived consumers born, where $L_t = (1+n)^t L_0$, with L_0 given and $n > 0$. At $t = 0$ there are some one-period-lived old agents who are collectively endowed with K_0 units of capital, and who maximize consumption at $t = 0$. A consumer born in period t has preferences given by

$$u(c_t^y, c_{t+1}^o) = \ln c_t^y + \ln c_{t+1}^o$$

where c_t^y (c_t^o) denotes consumption of a young (old) consumer in period t . Each consumer has one unit of time available when young. The production technology is given by

$$Y_t = K_t^\alpha L_t^{1-\alpha},$$

where Y_t is output, K_t is the capital input, L_t is the labor input, and $0 < \alpha < 1$. One unit of the consumption good can be converted into one unit of capital, and vice-versa. In period t , the government levies a lump-sum tax of τ_t^y on each young person, and a lump-sum tax of τ_t^o on each old person. The government can acquire capital in any period, with K_{t+1}^g denoting the capital acquired by the government in period t which becomes productive and is rented out by the government in period $t + 1$. Let w_t denote the real wage rate and r_t the capital rental rate.

- (a) Determine the capital/labor ratio, the consumption of the young, and the consumption of the old in a socially optimal steady state.
- (b) Write down the government's budget constraint.
- (c) Determine the optimal savings of a young consumer given market prices and taxes.

- (d) Suppose that the government introduces a pay-as-you-go social security system whereby transfers are made between the young and the old in each period, so that $\tau_t^y = -\tau_t^o = \tau$ and $K_{t+1}^g = 0$ for all t .
- Show that the government budget constraint is satisfied in each period.
 - Determine whether or not the government can set τ so as to achieve the socially optimal steady state in a competitive equilibrium.
 - If the answer to part (ii) is yes, determine the optimal level of τ and whether the optimal τ is positive or negative.
 - Explain your results.
- (e) Now suppose alternatively that the government introduces a fully-funded social security system, whereby $K_{t+1}^g = L_t\gamma$, $\tau_t^y = \gamma$, and $\tau_{t+1}^o = -\gamma(1 + r_{t+1})$ for $t = 0, 1, 2, \dots$.
- Show that the government budget constraint is satisfied in each period.
 - Determine whether or not the government can set γ so as to achieve a socially optimal steady state in a competitive equilibrium.
 - If the answer to part (ii) is yes, determine the optimal level of γ and whether the optimal γ is positive or negative.
 - Explain your results.
2. (35 points) The infinitely-lived representative consumer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t [\ln c_t + \gamma \ln l_t],$$

where $0 < \beta < 1$, c_t is consumption, l_t is leisure, and $\gamma > 0$. Each period, the consumer has one unit endowment of time which can be divided between work and leisure. The production technology is given by

$$y_t = k_t^\alpha n_t^{1-\alpha},$$

where y_t is output, k_t is the capital input, n_t is the labor input, and $0 < \alpha < 1$. Capital depreciates by 100% each period.

- Determine the paths followed by consumption, leisure, capital, and output, in a competitive equilibrium.
- How are your results in part (a) affected by γ ? Determine how γ affects the steady state levels of consumption, leisure, capital, and output. Explain your results.

3. (35 points) There is an infinite-lived representative consumer with preferences given by

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^\gamma}{\gamma},$$

where $0 < \beta < 1$, c_t is consumption, and $\gamma < 1$. The representative consumer has one unit endowment of time each period, which can be used for producing consumption goods, producing goods for the government, or producing human capital. The technology for producing consumption goods is given by

$$c_t = \alpha h_t u_t,$$

where $\alpha > 0$, h_t is the human capital of the representative consumer, and u_t is time spent by the consumer in the production of consumption goods. The consumer spends v_t units of time in each period working for the government, with production of government-consumed goods given by

$$g_t = \eta h_t v_t,$$

where $\eta > 0$. The government sets $v_t = v$, a constant, for $t = 0, 1, 2, \dots$. Human capital is produced according to

$$h_{t+1} = \delta h_t x_t,$$

where $\delta > 0$ and x_t is the quantity of time devoted to human capital accumulation.

- (a) Determine the paths followed by consumption, human capital, and total output over time, and determine how the consumer allocates time to the production of consumption goods and human capital accumulation in each period.
- (b) How are the growth rates of consumption and human capital, and the allocation of the consumer's time affected by a change in v ? Explain your results.