

**Economics 501**  
**Assignment 1**

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Due: September 12, 2006

1. Consider the following representative agent model. The consumer has preferences given by

$$u(c, \ell) = \alpha c + \ell,$$

where  $c$  is consumption,  $\ell$  is leisure, and  $\alpha > 0$ . The consumer is endowed with 1 unit of time and  $k_0$  units of capital. The representative firm has a technology for producing consumption goods, given by

$$y = zk^\gamma n^{1-\gamma},$$

where  $z$  is total factor productivity,  $k$  is the capital input,  $n$  is the labor input, and  $0 < \gamma < 1$ . Let  $w$  denote the wage rate and  $r$  the rental rate on capital.

- (a) Solve for competitive equilibrium prices and quantities (hint: there will be 2 cases to consider here, i.e. the solution will be quite different depending on whether  $\alpha z(1-\gamma)k_0^\gamma \leq 1$  or  $\alpha z(1-\gamma)k_0^\gamma > 1$ ). Drawing a picture should help.
  - (b) What are the effects of an increase in  $z$  on equilibrium prices and quantities? Explain your results.
2. Consider an economy where the representative consumer has a utility function  $u(c, \ell)$ , where  $c$  is consumption and  $\ell$  is leisure. The consumer has an endowment of 1 unit of time and  $k_0$  units of capital. The representative firm has a production technology given by

$$y = f(k, n),$$

where  $k$  is the capital input and  $n$  is the labor input. The government imposes a proportional tax on labor income, where the tax rate is  $t$ , with  $0 < t < 1$ . Revenue from the tax on labor income is rebated to the representative consumer lump-sum. Let  $\tau$  denote the lump sum transfer that the representative consumer receives.

- (a) Define a competitive equilibrium for this economy.
- (b) Show that the competitive equilibrium is not Pareto optimal.
- (c) How does the Pareto optimal allocation compare to the competitive equilibrium allocation? For example, is output larger or smaller, and is employment larger or smaller? Explain your results.