1. Consider the following representative agent model. The consumer has preferences given by
\[ u(c, \ell) = \alpha c + \ell, \]
where \( c \) is consumption, \( \ell \) is leisure, and \( \alpha > 0 \). The consumer is endowed with 1 unit of time and \( k_0 \) units of capital. The representative firm has a technology for producing consumption goods, given by
\[ y = zk^\gamma n^{1-\gamma}, \]
where \( z \) is total factor productivity, \( k \) is the capital input, \( n \) is the labor input, and \( 0 < \gamma < 1 \). Let \( w \) denote the wage rate and \( r \) the rental rate on capital.

(a) Solve for competitive equilibrium prices and quantities (hint: there will be 2 cases to consider here, i.e. the solution will be quite different depending on whether \( \alpha z(1-\gamma)k_0^\gamma \leq 1 \) or \( \alpha z(1-\gamma)k_0^\gamma > 1 \)). Drawing a picture should help.

(b) What are the effects of an increase in \( z \) on equilibrium prices and quantities? Explain your results.

2. Consider an economy where the representative consumer has a utility function \( u(c, \ell) \), where \( c \) is consumption and \( \ell \) is leisure. The consumer has an endowment of 1 unit of time and \( k_0 \) units of capital. The representative firm has a production technology given by
\[ y = f(k, n), \]
where \( k \) is the capital input and \( n \) is the labor input. The government imposes a proportional tax on labor income, where the tax rate is \( t \), with \( 0 < t < 1 \). Revenue from the tax on labor income is rebated to the representative consumer lump-sum. Let \( \tau \) denote the lump sum transfer that the representative consumer receives.
(a) Define a competitive equilibrium for this economy.

(b) Show that the competitive equilibrium is not Pareto optimal.

(c) How does the Pareto optimal allocation compare to the competitive equilibrium allocation? For example, is output larger or smaller, and is employment larger or smaller? Explain your results.