1. Consider the following representative agent model. The consumer’s utility function is \( u(c, l, g) \), where \( c \) is consumption, \( l \) is leisure, and \( g \) is the quantity of public goods provided by the government. The consumer has an endowment of one unit of time. The representative firm has a linear technology for producing consumption goods, given by

\[ y = n, \]

where \( n \) is the labor input in units of time, and \( y \) is output. The government levies a lump-sum tax \( \tau \) on the representative consumer (in units of consumption goods), and the government has a technology that converts consumption goods one-for-one into public goods, that is \( g = \tau \). Assume that all goods are normal, and that the utility function is strictly concave.

(a) Suppose first that the government sets \( g \) exogenously. Determine the effects of an increase in \( g \) on consumption, leisure, output, labor supply, and the real wage. How do your answers differ from the case in my notes where government spending simply involved the confiscation and destruction of goods? Explain.

(b) Now suppose that the government sets \( g \) so as to make the representative consumer as well off as possible. Derive a set of equations that determines \( c, l, \) and \( g \), and describe the economic intuition behind these equations.

2. Consider a representative agent model where the representative consumer has preferences given by the utility function \( u(c_1, c_2, l) \), where \( c_1 \) denotes consumption of market goods, \( c_2 \) denotes consumption of home-produced goods, and \( l \) is leisure. Assume that all goods are normal and that the utility function is strictly concave. The consumer has an endowment of one unit of time which can be supplied to the representative firm to produce market goods, used at
home to produce home goods, or taken as leisure. There is a representative firm that produces market goods according to

\[ y_1 = z_1 n_1, \]

where \( y_1 \) is output of market goods, \( z_1 \) is productivity in the market sector, and \( n_1 \) is labor input in the market sector. As well, the consumer has a technology for producing home goods, described by

\[ y_2 = z_2 n_2, \]

where \( y_2 \) is output of home goods, \( z_2 \) is productivity in the home sector, and \( n_2 \) is labor input in the home sector.

(a) What are the effects of an increase in \( z_1 \) on time spent working in the market, in home production, in leisure, and on consumption of home and market goods. Determine how your results depend on income and substitution effects.

(b) Repeat part (a) for an increase in \( z_2 \).

(c) Now, what happens if \( z_1 \) and \( z_2 \) increase by identical amounts? Explain your results.