

**Economics 501**  
**Assignment 3**

Steve Williamson  
September 19, 2006  
Due: September 26, 2006

1. Consider the following representative agent model. There is a representative consumer, with preferences given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t),$$

where  $\beta$  is the discount factor, with  $0 < \beta < 1$ ,  $c_t$  is consumption, and  $l_t$  is leisure. The government's budget constraint in period  $t$  is

$$g_t + (1 + r_t)b_t = \tau_t + b_{t+1},$$

for  $t = 0, 1, 2, \dots$ , where  $r_t$  is the real interest rate,  $b_t$  is the quantity of one-period bonds issued in period  $t - 1$  by the government, and  $\tau_t$  is a lump sum tax paid by the representative consumer. Assume that  $b_0 = 0$  and that  $g_t = 0$  for all  $t$ . Further, assume that

$$u(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} + l_t,$$

where  $\gamma > 0$ . The technology is given by  $y_t = z_t n_t$  where  $y_t$  is output,  $z_t$  is labor productivity, and  $n_t$  is labor input. Assume that productivity grows over time according to

$$z_t = z_0(1 + \alpha)^t,$$

where  $\alpha > -1$ , with  $z_0$  given. Assume throughout that in equilibrium  $l_t > 0$ .

- (a) Solve for a competitive equilibrium, determining  $\{y_t, c_t, l_t, n_t, w_t, r_{t+1}\}_{t=0}^{\infty}$ .
- (b) Show that consumption grows at a constant rate which depends only on  $\alpha$  and  $\gamma$ . What happens to the growth rate of consumption when  $\alpha$  or  $\gamma$  increases? Explain why you get these results.

- (c) Show that the real interest rate is constant, and depends only on  $\alpha$  and  $\gamma$ . Explain what you get when  $\alpha = 0$ . Determine what happens to the real interest rate when  $\alpha$  or  $\gamma$  changes. Explain your results, with reference to part (b).
2. Assume the same setup as in question 1, except that  $g_t = g_1$  for  $t = 0, 1, 2, 3, \dots, T$ , and  $g_t = g_2$  for  $t = T+1, T+2, T+3, \dots$ , where  $0 < g_2 < g_1$ . Determine the path followed by the real interest rate for  $t = 0, 1, 2, \dots$ , and explain your results.