1. Consider the following representative agent model. There is a representative consumer, with preferences given by
\[
\sum_{t=0}^{\infty} \beta^t u(c_t, l_t),
\]
where \( \beta \) is the discount factor, with \( 0 < \beta < 1 \), \( c_t \) is consumption, and \( l_t \) is leisure. The government’s budget constraint in period \( t \) is
\[
g_t + (1 + r_t)b_t = \tau_t + b_{t+1},
\]
for \( t = 0, 1, 2, ..., \) where \( r_t \) is the real interest rate, \( b_t \) is the quantity of one-period bonds issued in period \( t - 1 \) by the government, and \( \tau_t \) is a lump sum tax paid by the representative consumer. Assume that \( b_0 = 0 \) and that \( g_t = 0 \) for all \( t \). Further, assume that
\[
u(c_t) = \frac{c_t^{1-\gamma} - 1}{1-\gamma} + l_t,
\]
where \( \gamma > 0 \). The technology is given by \( y_t = z_t n_t \) where \( y_t \) is output, \( z_t \) is labor productivity, and \( n_t \) is labor input. Assume that productivity grows over time according to
\[
z_t = z_0 (1 + \alpha)^t,
\]
where \( \alpha > -1 \), with \( z_0 \) given. Assume throughout that in equilibrium \( l_t > 0 \).

(a) Solve for a competitive equilibrium, determining \( \{y_t, c_t, l_t, n_t, w_t, r_{t+1}\}_{t=0}^{\infty} \).

(b) Show that consumption grows at a constant rate which depends only on \( \alpha \) and \( \gamma \). What happens to the growth rate of consumption when \( \alpha \) or \( \gamma \) increases? Explain why you get these results.
(c) Show that the real interest rate is constant, and depends only on $\alpha$ and $\gamma$. Explain what you get when $\alpha = 0$. Determine what happens to the real interest rate when $\alpha$ or $\gamma$ changes. Explain your results, with reference to part (b).

2. Assume the same setup as in question 1, except that $g_t = g_1$ for $t = 0, 1, 2, 3, ..., T$, and $g_t = g_2$ for $t = T+1, T+2, T+3, ..., $ where $0 < g_2 < g_1$. Determine the path followed by the real interest rate for $t = 0, 1, 2, ...$, and explain your results.