

Economics 501
Assignment 4

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Due: October 17, 2006

1. Time is indexed by $t = 0, 1, 2, \dots, \infty$. In period t , L_t two-period-lived consumers are born, where

$$L_t = L_0(1 + n)^t,$$

with $L_0 > 0$. When young, each of these consumers is endowed with one unit of labor, and has preferences given by $u(c_t^y, c_{t+1}^o) = v(c_{t+1}^o)$, where $v(\cdot)$ is a strictly increasing function. There is a group of one-period-lived old agents alive at $t = 0$ who are collectively endowed with K_0 units of capital. These one-period-lived consumers maximize consumption at $t = 0$. The representative firm has a production technology given by $Y_t = K_t^\alpha L_t^{1-\alpha}$, where $0 < \alpha < 1$.

- (a) Determine consumption of the young, consumption of the old, and the capital/labor ratio in the optimal steady state.
 - (b) Determine consumption of the young, consumption of the old, and the capital/labor ratio in a competitive equilibrium steady state. How does this differ from the optimal steady state in part (a), and why?
 - (c) Now, suppose that the government issues B_{t+1} bonds in period t , where $B_{t+1} = bL_t$ for all t , with b a constant. Each young agent is taxed lump-sum so that the government can finance any interest payments on the debt that cannot be financed with the current bond issue. Determine the value for b that implies that an optimal steady state is achieved as a competitive equilibrium steady state. Is b positive or negative? Explain your results.
2. Assume the same setup as in problem 1, with the following exceptions. Let $Y_t = K_t^\alpha (z_t L_t)^{1-\alpha}$, where

$$z_t = z_0(1 + \rho)^t,$$

with $z_0 > 0$ and $\rho > -1$. That is, there is labor-augmenting technical change.

- (a) Determine a steady state. Here, it will help to note that $k_t \equiv \frac{K_t}{L_t z_t}$ converges to a constant in the steady state.
- (b) What are the growth rates of aggregate output, the aggregate capital stock, and aggregate consumption in the steady state?
- (c) What is happening to the consumption of each old person and the consumption of each young person in the steady state? What can we say about what is happening to the welfare of each generation in the steady state?
- (d) Explain your results, and discuss.