Economics 501
Assignment 5

Steve Williamson
September October 17, 2006
Due: October 24, 2006

1. The representative consumer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t \theta_t \log c_t,$$

where $0 < \beta < 1$, c_t is consumption, $\theta_t = \theta^*$ for $t = 0, 2, 4, ...$, and $\theta_t = \theta^{**}$ for $t = 1, 3, 5, ...$, where $\theta^* > \theta^{**}$. The production technology is given by

$$y_t = k_t,$$

where y_t is output and k_t is capital, with $k_0 > 0$ given. There is 100% depreciation, and the aggregate resource constraint is

$$c_t + i_t = y_t.$$

   (a) Let $v(k_t)$ and $w(k_t)$ denote the value functions in even periods and odd periods, respectively. Using guess-and-verify methods, determine $v(\cdot)$ and $w(\cdot)$, and the decision rules for consumption and investment.

   (b) This is a type of "cake-eating" problem, where it is as if k_0 is some initial quantity of food that the consumer has, food can be stored, and the consumer needs to determine how much to eat each period. He or she is more hungry in even periods than in odd periods. Explain your results in part (a) in light of this interpretation.

2. The representative consumer has preferences given by

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\gamma} - 1}{1 - \gamma} \right),$$

where $\gamma > 0$. The production technology is given by

$$y_t = \alpha k_t,$$

where $\alpha > 0$. Capital depreciates at the rate δ, with $0 < \delta < 1$. One unit of capital can be produced from one unit of the consumption goods in period t, and becomes productive in period $t + 1$.

(a) Assume that the value function is strictly concave and differentiable, and show that consumption grows at a constant rate. Determine what that rate is, and explain your results.

(b) Under what conditions will consumption increase without bound? Explain why this can happen here, whereas in the standard neoclassical growth model consumption converges to a constant if there is no technological change.