Economics 501
Assignment 6

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Due: October 31, 2006

1. Consider a model where the representative consumer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \log c_t,$$

where $0 < \beta < 1$ and c_t denotes consumption. The consumer is endowed with one unit of time each period. Output of consumption goods is produced according to the production technology

$$y_t = z_{1t} k_t^{\alpha} n_t^{1-\alpha},$$

where y_t denotes output, z_{1t} is total factor productivity, k_t is the capital input, and n_t is the labor input, with $0 < \alpha < 1$. In period t, it requires $\frac{1}{z_{2t}}$ units of consumption goods to produce one unit of capital. Capital produced in period t becomes productive at the beginning of period $t+1$, and then depreciates by 100%. Assume that (z_{1t}, z_{2t}) is a random draw in period t from a probability distribution $F(z_{1t}, z_{2t})$. That is, the current shocks are i.i.d., and we are allowing for the fact that z_{1t} and z_{2t} could be correlated.

   (a) Solve for decision rules for investment, consumption, and next period's capital stock.

   (b) What is the price of investment goods in a competitive equilibrium?

   (c) Suppose that z_{1t} and z_{2t} are uncorrelated. What are the model's predictions concerning the behavior of consumption and investment, and the propagation of shocks over time?

   (d) Now, suppose that $z_{2t} = \gamma z_{1t}$, with $\gamma > 0$, so that the shocks are perfectly positively correlated. How do your answers in part (c) change, and why?

2. Suppose an economy where the representative consumer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \theta_t u(c_t),$$

where $0 < \beta < 1$, θ_t is a random preference shock, and $u(\cdot)$ is a strictly increasing, strictly concave, and continuously differentiable utility function. The production technology is given by

$$y_t = \alpha k_t,$$

where y_t is output, k_t is the capital stock, and $\alpha > 0$. Assume that there is 100% depreciation of capital. Show that, in a competitive equilibrium, the marginal utility of consumption follows a martingale with multiplicative drift, and interpret this condition. [If a stochastic process x_t follows a martingale with multiplicative drift, then $E_t x_{t+1} = \gamma x_t$ for some constant $\gamma > 0$.]