

Economics 501
Assignment 7

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Due: November 21, 2006

1. Consider the following asset pricing model. There is a representative consumer who has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where $0 < \beta < 1$ and $u(\cdot)$ is a strictly concave, strictly increasing and twice continuously differentiable function. Aggregate output is produced by a single productive unit, where y_t is output in period t . Assume that y_t is an i.i.d. random variable. In period 0, the consumer has an endowment of one perfectly divisible share in the single productive unit. The share is traded on a competitive market each period, and is a claim to aggregate output each period. In addition, the consumer also trades bonds of different maturities. Each period, n different bonds are traded, with $i = 1, 2, \dots, n$, where i denotes the number of periods until the bond matures. In period t , an i -period bond is a claim to unit of of consumption in period $t + i$. A bond can be issued in any period, with any time to maturity up to n , and this bond will then be traded subsequently in competitive markets, in each period until it matures. Let p_t^i denote the price of a bond in period t that matures in period $t + i$. Then, let r_t^i denote the *yield to maturity* on an i -period bond, which is defined by

$$r_t^i = \left(\frac{1}{p_t^i} \right)^{\frac{1}{i}} - 1.$$

The *yield curve* is a graph of r_t^i as a function of i .

- (a) Derive a formula that determines p_t^i in terms of p_{t+1}^{i-1} .
- (b) Determine p_t^i in equilibrium.

- (c) For what value of y_t will the yield curve be flat? That is, under what conditions will we have $r_t^i = r$, a constant, for all i ? Determine r .
 - (d) How does the yield curve vary with the state y_t ? In what states of the world will the yield curve be upward-sloping? When will it be downward-sloping?
 - (e) Explain your results.
2. Assume a one-sided search model, identical to the one in my notes, but assume a particular wage distribution. That is, assume w is a draw from a uniform distribution on $[\underline{w}, \bar{w}]$, where $0 < \underline{w} < \bar{w}$.
- (a) Solve for the reservation wage w^* .
 - (b) Now suppose that \bar{w} increases, with the distribution still uniform over $[\underline{w}, \bar{w}]$. Determine the effects that this has on the mean wage offer, on the reservation wage, and on the probability that an unemployed worker receives an acceptable wage offer. Also, determine the effect on the steady state unemployment rate.
 - (c) Now, suppose that \underline{w} decreases by ρ , and \bar{w} increases by ρ , where $\rho > 0$. Determine the effects that this has on the mean wage offer, the variance of wage offers, the reservation wage, and the probability that an unemployed worker receives an acceptable wage offer. Again, determine the effect on the steady state unemployment rate.
 - (d) Explain your results in parts (b) and (c).