

Economics 501, Macroeconomics I
Final Exam Solution

STEVE WILLIAMSON

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1. Solutions are as follows.

(a) The Bellman equations, simplified, are

$$rV_e(w) = w + \delta[V_u^1 - V_e(w)], \quad (1)$$

$$rV_u^1 = b + \int_0^{\bar{w}} \max[V_e(w) - V_u^1, V_u^0 - V_u^1]f(w)dw, \quad (2)$$

$$rV_u^0 = \int_0^{\bar{w}} \max[V_e(w) - V_u^0, 0]f(w)dw \quad (3)$$

From equation (1), solve for $V_e(w)$ to get

$$V_e(w) = \frac{w + \delta V_u^1}{r + \delta}. \quad (4)$$

Therefore, since V_u^1 is a constant, $V_e(w)$ is linear and strictly increasing in w . Therefore, the optimization problems on the right-hand sides of (2) and (3) have the same solutions, i.e. an unemployed worker will accept any wage offer of $w \geq w^*$ and reject any offer of $w < w^*$, no matter how long he or she has been unemployed, where $V_e(w^*) = V_u^0$, where, from (4),

$$w^* = rV_u^0 - \delta(V_u^1 - V_u^0). \quad (5)$$

(b) We can now write equations (2) and (3) as

$$rV_u^1 = b + \int_{w^*}^{\bar{w}} V_e(w)f(w)dw - V_u^1 + F(w^*)V_u^0, \quad (6)$$

$$rV_u^0 = \int_{w^*}^{\bar{w}} V_e(w)f(w)dw - [1 - F(w^*)]V_u^0, \quad (7)$$

respectively. Therefore, subtracting (7) from (6), and solving for $V_u^1 - V_u^0$, we get

$$V_u^1 - V_u^0 = \frac{b}{1 + r} \quad (8)$$

Then, substituting in equation (5) for $V_u^1 - V_u^0$ using (8), we get

$$w^* = rV_u^0 - \frac{\delta b}{1 + r}. \quad (9)$$

Then, note that, from (4), and (5),

$$V_e(w) - V_u^0 = \frac{w - w^*}{r + \delta},$$

so we can write equation (7) as

$$rV_u^0 = \frac{1}{r + \delta} \int_{w^*}^{\bar{w}} (w - w^*)f(w)dw. \quad (10)$$

Then, use equation (9) to substitute for rV_u^0 in equation (10), which then simplifies as follows

$$w^* = -\frac{\delta b}{1 + r} + \frac{1}{r + \delta} \int_{w^*}^{\bar{w}} (w - w^*)f(w)dw,$$

then simplify the above equation further, as in my notes, using integration by parts, to get

$$w^* = -\frac{\delta b}{1 + r} + \frac{1}{r + \delta} \int_{w^*}^{\bar{w}} [1 - F(w)]f(w)dw. \quad (11)$$

Then, it is straightforward to show, by totally differentiating equation (11), that the reservation wage decreases when b increases. If we solve for the steady state unemployment rate u , as in my notes, we get

$$u = \frac{\delta}{\delta + 1 - F(w^*)},$$

and so, since w^* decreases with an increase in b , u will decrease when b increases. This all may seem odd, as it is typical in search models for the reservation wage to rise and the unemployment rate to increase when unemployment benefits rise. What is happening here is that, in the first period of unemployment after a separation, an unemployed worker has already received the unemployment insurance benefit b when he or she decides whether to accept the current wage offer. Usually, an increase in b will reduce the value of holding a job relative to the value of being unemployed, but here the opposite occurs. In this context the unemployment insurance payment is like a subsidy for working, since the worker has to get a job and be separated from it, before claiming unemployment insurance. Therefore, when b increases, this increases the value of working relative to the value of remaining unemployed, and the unemployed worker is therefore less picky about the job that he or she is willing to accept.

2. Solutions are as follows:

- (a) Asset prices are determined in the usual way (see my notes) by the stochastic Euler equations

$$p_{1t}u'(c_t) = \beta E_t[(\theta_t y + p_{i,t+1})u'(c_{t+1})], \quad (12)$$

$$p_{2t}u'(c_t) = \beta E_t\{(1 - \theta_t)y + p_{i,t+1}\}u'(c_{t+1}). \quad (13)$$

Now, in equilibrium, $c_t = y$ for all t , so $u'(c_t) = u'(y)$ for all t . Then, from (12) and (13) we get

$$p_{1t} = \beta E_t[\theta_t y + p_{i,t+1}] \quad (14)$$

$$p_{2t} = \beta E_t[(1 - \theta_t)y + p_{i,t+1}] \quad (15)$$

Now, since θ_t is an i.i.d. random variable and p_{it} depends only on θ_t for each i , the terms inside the expectation operators in (14) and (15) are i.i.d. random variables, which implies that $p_{1t} = p_1$ and $p_{2t} = p_2$ for all t , where p_1 and p_2 are constants. Then, solving for p_1 and p_2 using equations (14) and (15), we get

$$p_1 = \frac{\beta y E[\theta]}{1 - \beta}, \quad (16)$$

$$p_2 = \frac{\beta y \{1 - E[\theta]\}}{1 - \beta} \quad (17)$$

- (b) It is straightforward to show that the price of the risk-free bond is $q_t = \beta$ for all t .
- (c) From (16) and (17), the expected gross returns on shares 1 and 2 are, respectively,

$$E_t \left[\frac{p_{1,t+1} + \theta_{t+1} y}{p_{1t}} \right] = 1 + \frac{y E[\theta]}{\frac{\beta y E[\theta]}{1 - \beta}} = \frac{1}{\beta},$$

$$E_t \left[\frac{p_{2,t+1} + (1 - \theta_{t+1}) y}{p_{2t}} \right] = 1 + \frac{y \{1 - E[\theta]\}}{\frac{\beta y \{1 - E[\theta]\}}{1 - \beta}} = \frac{1}{\beta}.$$

Since the gross rate of return on the risk-free bond is also $\frac{1}{\beta}$, therefore the risk premia on the two shares are zero, since all three assets have an expected rate of return of $\frac{1}{\beta} - 1$ in each period. We get this result because there is no aggregate risk in this economy. Aggregate output and aggregate consumption are constants, and the marginal utility of consumption is constant at $u'(y)$ for all t . Shares are subject only to idiosyncratic risk, but when both shares are held the consumer gets perfect diversification, since the dividends on the shares are perfectly negatively correlated. Therefore, all assets are priced so that they bear the same expected rates of return, equal to the subjective discount rate.

3. The representative consumer faces the cash-in-advance constraint

$$P_t c_t + S_t B_{t+1} + Q_t z_{t+1} \leq M_t + B_t + Q_t z_t + P_t \tau_t, \quad (18)$$

and the budget constraint

$$P_t c_t + S_t B_{t+1} + Q_t z_{t+1} + M_{t+1} = M_t + B_t + Q_t z_t + P_t \tau_t + P_t y_t z_{t+1}, \quad (19)$$

where $P_t y_t z_{t+1}$ is total dividends received from the firm. Letting λ_t denote the multiplier associated with the cash-in-advance constraint, and μ_t the multiplier associated with the budget constraint, the first order necessary conditions for an optimum are

$$u'(c_t) - P_t(\lambda_t + \mu_t) = 0, \quad (20)$$

$$-Q_t(\lambda_t + \mu_t) + P_t y_t \mu_t + \beta E_t [Q_{t+1}(\lambda_{t+1} + \mu_{t+1})] = 0, \quad (21)$$

$$-S_t(\lambda_t + \mu_t) + \beta E_t(\lambda_{t+1} + \mu_{t+1}) = 0, \quad (22)$$

$$-\mu_t + \beta E_t(\lambda_{t+1} + \mu_{t+1}) = 0. \quad (23)$$

Then, using (20) and (22), respectively, to substitute for $\lambda_t + \mu_t$ and μ_t in equations (21) and (22), we get

$$\frac{Q_t u'(c_t)}{P_t} = P_t y_t \beta E_t \left[\frac{u'(c_{t+1})}{P_{t+1}} \right] + \beta E_t \left[Q_{t+1} \frac{u'(c_{t+1})}{P_{t+1}} \right], \quad (24)$$

$$S_t \frac{u'(c_t)}{P_t} = \beta E_t \left[\frac{u'(c_{t+1})}{P_{t+1}} \right] = 0. \quad (25)$$

Our equilibrium conditions are $c_t = y_t$, $\bar{M}_t = M_t$, $B_t = 0$, and $z_t = 1$ for all t . Then, a binding cash-in-advance constraint gives

$$P_t = \frac{\bar{M}_{t+1}}{y_t}, \quad (26)$$

which is the equilibrium solution for the price level. That is, the price level increases in proportion to the money stock (after the transfer) in the current period. Note that changes in the money supply can have no real effects here, since output is exogenous. Given a binding cash-in-advance constraint, all money is spent on the available supply of output, and so the price level must be proportional to the money supply. Now, substituting for P_t in equations (24) and (25) using (26), we get

$$\frac{Q_t u'(y_t) y_t}{\bar{M}_{t+1}} = \bar{M}_{t+1} \beta E_t \left[\frac{u'(y_{t+1}) y_{t+1}}{\bar{M}_{t+2}} \right] + \beta E_t \left[Q_{t+1} \frac{u'(y_{t+1}) y_{t+1}}{\bar{M}_{t+2}} \right], \quad (27)$$

$$S_t \frac{u'(y_t) y_t}{\bar{M}_{t+1}} = \beta E_t \left[\frac{u'(y_{t+1}) y_{t+1}}{\bar{M}_{t+2}} \right] = 0. \quad (28)$$

In equations (27) and (28) in each case where there is an expectation, this is the expectation of an i.i.d. random variable. Therefore, we can say that S_t increases in proportion to \bar{M}_{t+1} , that is a surprise increase in the money supply causes the nominal interest rate to fall. This is because, when the current money supply is high, the expected inflation rate is low. That is, from (26), the expected gross rate of inflation is

$$E_t \left[\frac{P_{t+1}}{P_t} \right] = \frac{y_t}{\bar{M}_{t+1}} E_t \left[\frac{\bar{M}_{t+2}}{y_{t+1}} \right],$$

which is decreasing in \bar{M}_{t+1} . The nominal interest rate falls because of a Fisher effect. From equation (27), Q_t rises more than in proportion to an increase in \bar{M}_{t+1} . This is because the dividend that is received during the current period (in cash) cannot be spent until the next period. When the current money supply is high, this means that the consumer expects the inflation rate to be low, and this then increases the real value of the dividend in terms of its purchasing power in the next period.