Economics 501, Macroeconomics I
Test 1

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Instructions: Read the questions carefully, and make sure to show all your work. Feel free to make any extra assumptions that you deem appropriate to get results. You have 2 hours. Good luck!

1. [40 points] Consider the following one-period competitive equilibrium representative agent model. There is a representative consumer, who has a utility function $u(c, l)$, where c is consumption and l is leisure. The market real wage is w, and the rental rate on capital is r. The consumer is endowed with one unit of time and k_0 units of capital. The representative firm has a production technology given by

$$y = \alpha(k + g) + n,$$

where y is output of consumption goods, k is the capital input for the firm, g is the quantity of public goods provided by the government, n is the labor input, and $0 < \alpha < 1$. The firm chooses its capital input and labor input treating g as given. The government levies a lump-sum tax τ on the consumer in order to finance public goods expenditure, with $g = \tau$. Public goods are like roads and bridges, i.e. they are public capital that generates private output.

   (a) Assume that g is exogenous. Define a competitive equilibrium, and derive a set of equations that solve for output, consumption, leisure, labor supply, the real wage, and the rental rate on capital in a competitive equilibrium.

   (b) Again with g exogenous, determine the effects of an increase in g on output, consumption, leisure, labor supply, the real wage, and the rental rate on capital. Carefully explain your results, and what they depend on. In particular, how does the value of α matter, and why?

   (c) Now, suppose that the government sets g in order to maximize the welfare of the representative consumer in a competitive equilibrium. Determine the optimal level of g, and explain your results.

2. [40 points] Consider the following one-period competitive equilibrium representative agent model. There is a representative consumer, who has a utility function $u(c, l)$, where c is consumption and l is leisure. The market real wage is w. The consumer is endowed with one unit of time. The representative firm has a production technology given by

$$y = z(l)n,$$

where y is the firm's output, n is the labor input chosen by the firm, and $z(l)$ is the firm's productivity. When the firm chooses n it treats l as being given. Assume that $z(0) = \bar{z} > 0$, $z(l^*) = 0$ for some l^* with $0 < l^* < 1$, $z'(l) < 0$ for $0 \leq l \leq l^*$,and $z''(l)(1 - l) - 2z'(l) < 0$ for $0 \leq l \leq l^*$. What this is intended to capture is that there is a negative externality on production associated with leisure activity by the consumer. The more leisure the consumer takes, the lower is aggregate productivity because, for example, people taking leisure clog the transportation network and make it difficult to do business.

(a) Derive the production possibilities frontier for this economy, and draw a diagram of it.

(b) Determine a Pareto optimum for this economy, and determine a competitive equilibrium, and show both in your diagram.

(c) Are the Pareto optimum and the competitive equilibrium the same, or are they different? Explain why they are the same or why they are different. If they are different, then determine how consumption and leisure differ at the Pareto optimum from what they are in a competitive equilibrium.

(d) Explain your results.

3. [40 points] Consider the following representative-agent infinite-horizon economy. There is a consumer with preferences given by

$$\sum_{t=0}^{\infty} \beta^t (\alpha \log c_t + \log l_t),$$

where $\alpha > 0$, $0 < \beta < 1$, c_t is consumption, and l_t is leisure. The consumer has an endowment of one unit of time each period. The representative firm has a production technology given by

$$y_t = z_t n_t,$$

where y_t is output, z_t is exogenous productivity, and n_t is the labor input. The consumer sells labor to the firm in period t at the price w_t, and there is a bond market on which the government and the consumer trade one-period bonds. A bond issued in period t exchanges for one unit of the period-t consumption good, and is a claim to $1 + r_{t+1}$ units of consumption in period $t + 1$. Assume that the consumer and the government each have zero bonds outstanding at the beginning of period 0. The government levies a lump-sum tax τ_t on the consumer in period t, and government spending in each period is assumed to be zero. Assume that $z_t = z^* > 0$ for $t = 0, 1, 2, 3, ..., T - 1, T + 1, T + 2, T + 3, ...,$ and $z_t = \gamma z^*$ for $t = T$, where $\gamma > 1$.

(a) Write down the consumer's budget constraint in period t, letting s_{t+1} denote the bonds purchased by the consumer in period t. Also write down the consumer's intertemporal budget constraint (you needn't derive it if you can remember what it is).

(b) Write down the government's budget constraint in period t, and the government's intertemporal budget constraint that must hold in equilibrium.

(c) Now, assume that $\tau_t = 0$ for $t = 0, 1, 2, ...$. Solve for $\{c_t, l_t, n_t, y_t, w_t, r_{t+1}\}_{t=0}^{\infty}$ in a competitive equilibrium, and explain your results.

(d) Alternatively suppose that the government, seeing that productivity will be temporarily high in period T, decides to increase taxes temporarily. That is, it sets $\tau_T = \tau^* > 0$, and sets $\tau_t = \tau^{**}$ for $t \neq T$. Again, solve for a competitive equilibrium as in part (c), and solve for τ^{**} (if you can't remember how to determine the sum of a geometric series, don't worry about it).