Economics 501, Macroeconomics I
Test 2

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Instructions: Read the questions carefully, and make sure to show all your work. You have 2 hours. Good luck!

1. [60 points] Time is indexed by $t = 0, 1, 2, ..., \infty$. There are L_t two-period-lived consumers born in period t, and $L_t = (1+n)L_{t-1}$ for $t = 1, 2, 3, ...$, with L_0 given. In period 0 there is a group of one-period-lived old consumers who are collectively endowed with K_0 units of capital, and who maximize period 0 consumption. Each young consumer in period t is endowed with one unit of time, and maximizes $u(c_{t+1}^o)$, where $u(\cdot)$ is a strictly increasing function, and c_{t+1}^o is consumption in old age. There is a representative firm that hires labor L_t each period, rents capital, K_t, and produces output according to

$$Y_t = F(K_t, L_t),$$

where $F(\cdot, \cdot)$ is a production function which is increasing in both arguments and is homogeneous of degree one. Consumption goods can be converted one-for-one into capital at any time, and capital can be converted one-for-one back into consumption goods. Capital produced in period t does not become productive until period $t+1$, and capital depreciates at the rate δ when it is used in production. That is, if there is K_t units of capital used in the production process at the beginning of period t, only $(1-\delta)K_t$ units remain at the end of the period. There is a government that levies a lump-sum tax τ_t on each young consumer in period t, and uses the proceeds to pay a lump-sum transfer ϕ_t to each old consumer in period t. The government does not borrow or lend.

   (a) Determine the characteristics of an optimal steady state.

   (b) Write down the government's budget constraint.

   (c) Determine a young consumer's optimal savings, consumption when young, and consumption when old, as functions of w_t, r_{t+1}, τ_t, and ϕ_{t+1}, where w_t and r_t are the real wage and the capital rental rate, respectively, in period t.

   (d) Derive a difference equation that (in principle) solves for the sequence of competitive equilibrium capital-labor ratios $\{k_{t+1}\}_{t=0}^{\infty}$ given a government policy $\{\tau_t, \phi_t\}_{t=0}^{\infty}$ that satisfies the government budget constraint in each period t.

   (e) Now, suppose that the government sets $\tau_t = \tau$ for all t and $\phi_t = \phi$ for all t, and show that τ and ϕ can be set so as to achieve the optimal steady state you determined in part (a), as a competitive equilibrium. Derive a set of equations that solves for the optimal τ and the optimal ϕ.

   (f) What is government policy accomplishing here? Discuss why this policy works to achieve an optimal steady state as a competitive equilibrium.

2. [60 points] Consider the following representative agent model. The representative consumer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \theta_t \ln c_t,$$

where $0 < \beta < 1$ and $\{\theta_t\}_{t=0}^{\infty}$ is known in period 0, where

$$\{\theta_t\}_{t=0}^{\infty} = \{\theta^*, \theta^{**}, \theta^*, \theta^{**}, ...\},$$

where $\theta^* > \theta^{**}$. The production technology is given by

$$y_t = z_t k_t,$$

where y_t is output, k_t is the capital stock, and z_t is an i.i.d. random variable, with $k_0 > 0$ given. Assume that capital depreciates by 100% each period.

(a) Write down the dynamic program, the solution to which is a competitive equilibrium.

(b) Solve the dynamic programming problem to determine optimal decision rules for consumption and investment.

(c) What does your solution in part (b) tell you about the behavior of consumption and investment in response to θ_t and z_t? Discuss.

(d) Calculate

$$E_t[\ln y_{t+1} - \ln y_t],$$

which is approximately the expected rate of growth in output between period t and period $t+1$, conditional on period t information. What does this depend on? Explain.