COSTLY MONITORING, FINANCIAL INTERMEDIATION, AND EQUILIBRIUM CREDIT RATIONING

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This paper establishes a link between equilibrium credit rationing and financial intermediation, in a model with asymmetrically informed lenders and borrowers, costly monitoring, and investment project indivisibilities. Intermediation is shown to dominate borrowing and lending between individuals, and these financial intermediaries exhibit several of the important features of intermediaries as we know them. Equilibrium interest rates and the aggregate quantity of loans respond quite differently to changes in taste and technology parameters, depending on whether or not there is rationing in equilibrium.

1. Introduction

The purpose of this paper is to analyze an environment with asymmetrically informed borrowers and lenders which for our purposes has two features of primary interest: (1) financial intermediation arises endogenously as the dominant vehicle for borrowing and lending in equilibrium, and (2) an equilibrium may exhibit credit rationing. In the model, the costly monitoring of lenders by borrowers and large-scale investment projects imply that there exist increasing returns to scale in lending and borrowing which can be exploited by financial intermediaries. Costly monitoring and universal risk neutrality yield debt contracts as optimal arrangements between lending institutions and borrowers. This in turn generates an asymmetry in these agents' payoff functions, which then permits equilibria with credit rationing. All borrowers are identical, ex ante, but in equilibrium some of these borrowers may receive loans while others do not. The system behaves quite differently in response to changes in underlying parameters which characterize technology and preferences, depending on whether or not there is credit rationing in equilibrium.

There are two literatures which are directly related to this paper, both of which study the financial market implications of informational asymmetries. First is the literature which has attempted to explain credit rationing as an

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Debt contracts are also derived as the real option to a bilateral contracting problem in equilibrium. The outcomes of default and non-default, which are the two extreme points, are determined by market forces in equilibrium. These outcomes are generated by the interactions of the borrower and the lender in the market, and they reflect the economic fundamentals of the underlying asset. In the context of this model, the borrower must take into account the potential for default and the associated costs, while the lender must consider the potential for returns and the risk of default.

The credit rationing literature, which has been developed over the years, discusses the role of the contract in determining the level of credit. The literature is divided into two main streams: the credit rationing literature and the financial market literature. The credit rationing literature focuses on the role of credit in determining the level of economic activity, while the financial market literature focuses on the role of credit in determining the allocation of resources.

In our model, the contract is derived from the underlying asset, and it is designed to reflect the economic fundamentals of the asset. The contract is designed to allow the borrower to benefit from the upside potential of the asset, while at the same time limiting the potential for default.

The optimal contract is determined by the market forces in equilibrium. The contract is designed to reflect the underlying asset, and it is determined by the interaction of the borrower and the lender in the market. The contract is designed to allow the borrower to benefit from the upside potential of the asset, while at the same time limiting the potential for default.
there is no rationing. However, if there is rationing, interest rates will remain unchanged, but the number of borrowers who do not receive loans will change. As in Williamson (1984b), this is consistent with the thrust of the availability doctrine [see Roosa (1951)] in that monetary policy (which could affect alternative rates of return in our model) can have real effects without changing interest rates in lending markets.

The remainder of the paper is organized as follows. In section 2 we present the model. Section 3 contains an examination of a regime with direct lending where intermediation is prohibited. We show that debt contracts are optimal and define an equilibrium. In section 4 we study the features of a regime with financial intermediation. Again, debt contracts are optimal. We define an equilibrium, and discuss existence and uniqueness. We show that intermediation dominates direct lending and that credit rationing may be a feature of the equilibrium. Section 5 contains a discussion of some comparative statics experiments. The final section is a summary and conclusion.

2. The model

This model is a version of that presented in Williamson (1984b), with modifications designed to permit a role for financial intermediation.

There are two periods, period zero, the planning period, and period one, when consumption takes place. There exists a single consumption good, which is perishable between period zero and period one.

There is a countable infinity of agents, each of whom is either a lender or an entrepreneur. We specify the population in terms of the probabilities of drawing agents of each type from the population, and write equilibrium conditions for the economy as a whole in per capita terms. If we draw an agent at random from the population, then

\[ \Pr[\text{agent is a lender}] = \alpha, \]
\[ \Pr[\text{agent is an entrepreneur}] = 1 - \alpha, \]

where \(0 < \alpha < 1\).

Each lender is endowed with a single indivisible unit of the consumption good in period zero which may be lent to entrepreneurs (directly or indirectly) or invested at a certain rate of return. To generate an upward-sloping supply curve for loanable funds, different lenders face different certain rates of return. That is, if an individual lender invests \(x\) units of the consumption good in period zero, she receives a return of \(s\) units of the consumption good in period

\[ s = \begin{cases} tx, & 0 \leq x \leq 1, \\ t, & x \geq 1. \end{cases} \]

I.e., certain return investment projects have a capacity of one unit of input. We have \(t \in [t, t]\), with \(0 < t < t < \infty\). If we were to draw an agent at random, then

\[ \Pr[t < t' | \text{agent is a lender}] = \int_t^{t'} h(t') \, dt, \quad t' \geq t, \]

where \(h(\cdot)\) is a probability density function which is positive and continuous on \([t, t]\).

Each entrepreneur receives a zero endowment in period zero, and has access to an investment project which yields a random return of \(K\) units of the consumption good in period one to an input of \(K\) units in period zero, and zero units otherwise. Here, \(K\) is an integer with \(K \geq 2\), and \(w\) is a random variable. Project returns are independent across entrepreneurs and, for each entrepreneur, \(w\) is distributed according to the probability density function \(f(\cdot)\) and probability distribution function \(F(\cdot)\). The function \(f(\cdot)\) is positive and differentiable on \([0, w]\), where \(w > 0\), and is zero otherwise. We have \(\alpha > (1 - \alpha)K\), so that the demand for credit is at least potentially satisfied.

The realization of \(w\), denoted \(w\), is costlessly observable only to the entrepreneur, but all agents know \(f(\cdot)\). In period one, an individual lender can expend effort to learn the return(s) on any project(s). It requires \(c\) units of effort to observe the return on any one project, where \(e > 0\). Lenders are endowed with an unbounded quantity of effort and maximize the expected value of \(U(x, e)\), where \(x\) is period one consumption and \(e\) is effort with \(x, e \geq 0\). We have

\[ U(x, e) = x - ce. \tag{2.1} \]

Entrepreneurs receive an endowment of zero units of effort and are risk-neutral with respect to consumption realizations. Entrepreneurs maximize the expected value of \(V(x)\) where \(x\) is consumption and \(x \geq 0\). We have

\[ V(x) = x. \tag{2.2} \]

Two important features of the model are the nature of the informational asymmetry and the timing of monitoring decisions. As in Gale and Hellwig (1984), Townsend (1979) and Diamond (1984), agents are asymmetrically informed (in the absence of monitoring) ex post. This contrasts with the environments of Boyd and Prescott (1985) and Stiglitz and Weiss (1981),
which feature ex ante information asymmetries. In our model, as in Diamond (1984), costly monitoring generates a role for financial intermediation. However, our monitoring technology is different from Diamond’s, where monitoring decisions are made ex ante, and similar to that of Gale and Hellwig (1984) and Townsend (1979), where monitoring decisions are made ex post. The fact that agents make monitoring decisions conditional on observations in period one will be seen to be crucial in permitting equilibria with financial intermediation and credit rationing.

In our model and those of Boyd and Prescott (1985) and Diamond (1984), a role for intermediation arises in spite of the fact that all agents are risk-neutral with respect to consumption realizations. Gale and Hellwig (1984) and Stiglitz and Weiss (1981) also restrict their analyses to models with risk-neutral agents, though these authors do not study financial intermediation. As in Gale and Hellwig (1984), risk neutrality simplifies the contracting problem considerably.

3. Equilibrium with direct lending

We first look at a regime where intermediation is prohibited, so that the only contractual arrangements are those between individual lenders and entrepreneurs.

Let \( r \) denote the certain market return, which is an endogenous variable, but which is treated as a fixed parameter by all agents.\(^4\) First, consider a single entrepreneur who wishes to fund her project. This entrepreneur must then offer contracts to \( K \) lenders. Without loss of generality, we assume that these contracts must be identical for each lender. Contracts must specify that one unit of the consumption good will be transferred from each lender to the entrepreneur in period zero, and will also specify the states of the world in which monitoring will occur and the payment schedules in the cases in which monitoring occurs and in which it does not. We consider only pure strategy contracts, i.e., attention is restricted to non-stochastic monitoring.\(^5\)

Following the realization of \( \tilde{w} \), the entrepreneur emits a signal, \( w^d \), to the \( K \) lenders, where \( w^d \in [0, \tilde{w}] \). The contract specifies that, if \( w^d \in S \subset [0, \tilde{w}] \), then monitoring occurs, while if \( w^d \not\in S \), then monitoring does not occur. The payment to each lender will then be

\[
R = R(w), \quad w^d \in S, \quad K(w^d), \quad w^d \not\in S,
\]

where \( R(\cdot) \) and \( K(\cdot) \) are functions which must obey the following feasibility conditions

\[
0 \leq R(w) \leq w, \quad 0 \leq K(w^d) \leq w, \tag{3.1}
\]

If the entrepreneur chooses \( w^d \not\in S \), then clearly she will always choose \( w^d = \arg \min_{w^d} K(w^d) \). Therefore, when monitoring does not occur the payment the entrepreneur makes to each lender is a constant, denoted by

\[
\bar{R} = \min_{w^d}(K(w^d)).
\]

It remains to determine the optimal payment schedule when monitoring occurs, \( R(w) \). This payment schedule must be incentive compatible, i.e., we must have

\[
w^d \in S \quad \text{if} \quad R(w) < \bar{R}, \tag{3.3}
\]

\[
w^d = w^d^* \quad \text{if} \quad R(w) > \bar{R}, \tag{3.4}
\]

and

\[
w^d \in S \quad \text{or} \quad w^d = w^d^* \quad \text{if} \quad R(w) = \bar{R}. \tag{3.5}
\]

Conditions (3.3) - (3.5) allow us to determine the realizations of \( \tilde{w} \) over which monitoring occurs, given \( R(w) \). Let \( B \) and \( B^c \) be non-intersecting subsets of \([0, \tilde{w}]\), with \( B + B^c = [0, \tilde{w}] \). \( B - \{ w : R(w) < \bar{R} \} \) and \( B^c - \{ w : R(w) \geq \bar{R} \} \). I.e., for \( w \in B \), monitoring occurs, and for \( w \in B^c \), monitoring does not occur. The optimal contract is a payment schedule - 'interest rate' pair \((R(w), \bar{R})\) which maximizes the entrepreneur’s expected utility, while giving each lender a level of expected utility of at least \( r \), the market interest rate:

\[
\max_{(R(w), \bar{R})} \left\{ \int_B [R(w) - \bar{R}] f(w) \, dw + \int_{B^c} \bar{R} f(w) \, dw \right\}, \tag{3.6}
\]

subject to

\[
\int_B \left[ R(w) - c \right] f(w) \, dw + \int_{B^c} \bar{R} f(w) \, dw \geq r.
\]

First, note that the constraint in (3.6) must be binding, since otherwise \( R(w) \)
and $\bar{R}$ could decrease, with the sets $B$ and $B'$ remaining unchanged, the constraint would still be satisfied, and the value of the objective function would increase.

**Proposition 1.** The optimal payment schedule is $R(w) = w$.

**Proof.** Suppose not, and that $(R'(w), \bar{R})$ is the optimal contract. Let $B' = \{ w : R'(w) < \bar{R} \}$ and $B'' = \{ w : R'(w) \geq \bar{R} \}$.

Then, from (3.6),

$$\int_{B'} [R'(w) - c] f(w) \, dw + \int_{B''} \bar{R} f(w) \, dw = r.$$

Now consider another payment schedule $R''(w)$ with $R''(w) \geq R'(w)$ for all $w \in [0, \bar{w}]$, $R''(w) > R'(w)$ for some $w \in B'$, and $R''(\cdot)$ continuous and monotone increasing on $[0, \bar{w}]$. Then, there is some $\tilde{R}''$ with $0 < \tilde{R}'' < \bar{R}$ such that

$$\int_{B'} [R''(w) - c] f(w) \, dw + \int_{B''} \tilde{R}'' f(w) \, dw = r,$$

$$B'' = \{ w : R''(w) < \tilde{R}'' \}, \quad B''' = \{ w : R''(w) \geq \tilde{R}'' \}.$$

The change in the objective function in (3.6) as the result of changing the contract from $(R'(w), \bar{R})$ to $(R''(w), \tilde{R}'')$ is

$$Kc \left[ \int_{B'} f(w) \, dw - \int_{B''} f(w) \, dw \right] > 0,$$

as $B'' \subset B'$ and $B' - B'' + \phi$. We therefore have a contradiction. Q.E.D.

The proof of Proposition 1 is adapted from Williamson (1984b) and a proof of a similar proposition in a different environment (which is in some ways more general) is in Gale and Hellwig (1984). Proposition 1 states that the optimal contract is what Gale and Hellwig call a 'standard debt contract', i.e., either each lender receives a fixed payment, $\bar{R}$, or the entrepreneur defaults, each lender monitors, and the entrepreneur receives a zero return (bankruptcy). The 'interest rate', $\bar{R}$, is then sufficient to characterize the contract, where $\bar{R}$ solves

$$\max_{\bar{R}} K \int_{\bar{R}}^{\tilde{R}} (w - \bar{R}) f(w) \, dw,$$

subject to

$$\int_{0}^{\bar{R}} wf(w) \, dw + \bar{R} (1 - F(\bar{R})) - c F(\bar{R}) = r.$$

Assuming that the following condition holds for $0 \leq \bar{R} \leq \bar{w}$:

$$f(\bar{R}) + c f'(\bar{R}) > 0,$$

then the expected utility of each lender is a concave function of $\bar{R}$, reaching a maximum for some $\bar{R} \in [0, \bar{w}]$, while the expected utility of the entrepreneur is decreasing in $\bar{R}$. As in Stiglitz and Weiss (1981) and as we discuss in more detail in Williamson (1984b), the asymmetry in the payoff functions of lenders and entrepreneurs leads to the possibility of credit rationing in equilibrium. In contrast to Stiglitz and Weiss (1981), this asymmetry did not come about due to moral hazard or adverse selection, but the form of the contract is derived rather than assumed. Debt contracts are also optimal in the environments specified by Gale and Hellwig (1984) and Diamond (1984), though in Diamond's paper bankruptcy does not impose costs of monitoring on the lender, but instead implies non-pecuniary costs for the borrower.

**Definition 1.** An equilibrium is a loan interest rate $\bar{R}^*$, a certain market interest rate $r^*$ and an aggregate loan quantity $q^*$, which satisfy:

1) $\bar{R}^*$ solves (3.7) given $r - r^*$.
2) $q^* - a \int_{t_0}^{t_1} h(t) \, dt$.
3) Either (a) $q^* = (1 - a)K$, or (b) $q^* < (1 - a)K$ and $1 - F(\bar{R}^*) - c F(\bar{R}^*) = 0$.

There are two types of equilibria, those with rationing ( RA equilibria) given by 3b) and those without rationing (NRA equilibria) given by 3a). In an RA equilibrium, all entrepreneurs offer the same contracts on the market, while lenders who wish to accept one of these contracts choose an entrepreneur at random. If all lenders have chosen an entrepreneur, a given entrepreneur is paired with a positive number of lenders, but this number is insufficient to fund the project, then these lenders choose another entrepreneur at random. This process continues until all lenders are allocated to entrepreneurs and any projects that are funded are fully funded. It is then possible for some lenders to change their offer to the entrepreneur. Indeed, monitoring is necessary since there is otherwise an incentive for the borrower to misreport his project return. However, moral hazard enters in a very different manner in the Stiglitz-Weiss paper.

*Actually, it is not quite correct to say that moral hazard is not an important element here. Indeed, monitoring is necessary since there is otherwise an incentive for the borrower to misreport his project return. However, moral hazard enters in a very different manner in the Stiglitz-Weiss paper.*
entrepreneurs not to receive loans, since if 3b) holds there is no contract which a rationed entrepreneur can offer which will bid loans away from other entrepreneurs, or draw additional lenders into the market.

4. Financial intermediation

In both the RA and NRA equilibria of the previous section, duplication of effort occurs, in that each entrepreneur borrows from \( K \) lenders, who each monitor the entrepreneur when default occurs. Potentially, a group of lenders could act as individual monitoring agents or auditors, who monitor entrepreneurs and sell the information to lenders, thus exploiting the economies of scale in monitoring entrepreneurs. However, such a market for information would fail since the value of information does not diminish with use. Also, the information may not be reliable due to incentives to cheat on the part of the auditors, and because entrepreneurs have an incentive to make side-payments to auditors to induce them to reveal incorrect information [see Campbell and Kracaw (1980)]. These problems can be solved, however, if a proportion of lenders act as intermediaries. Each intermediary is an individual lender, who acts to maximize expected utility by issuing financial claims to other lenders (depositors) and lending \( K \) units of the consumption good to each entrepreneur she contracts with. One entrepreneur is then matched with one lending institution, though an intermediary may lend to more than one entrepreneur. The expected utility cost of monitoring entrepreneurs is covered by the expected return on the intermediary's portfolio, i.e., the value of information is captured in a private good [see Leland and Pyle (1977)]. As in Diamond (1984), there remains the problem that the intermediary's depositors will need to monitor the intermediary when it defaults. By holding a diversified portfolio, the intermediary can reduce or eliminate (with a large number of independent risks) the expected utility loss to depositors due to monitoring.

Though the monitoring technology and the timing of monitoring decisions are different in this model than in Diamond (1984), the way we model intermediation is quite similar. Intermediaries are single agents, though we do not identify separate intermediary agents, as does Diamond, but instead allow lenders to function as intermediaries, as depositors, or as investors in projects with certain returns. In contrast to the approach of modelling intermediaries as single agents, intermediation in Boyd and Prescott (1985) is performed by multi-agent coalitions. This approach is tractable in their framework, as the production of information is assumed to be public.

Suppose that a given intermediary contracts to fund \( m \) entrepreneurs, indexed by \( j = 1, 2, \ldots, m \), that it contracts with \( mK - 1 \) lenders to act as 'depositors', and that it invests its single unit of the consumption good in the intermediary. Without loss of generality, assume that the intermediary writes identical contracts with each of the (ex ante) identical entrepreneurs. As in the regime with contracting between individuals, entrepreneur \( j \) pays the intermediary a gross rate of return of \( \hat{R} \) in the event that monitoring does not occur. Let \( R(w) \) denote the payment per unit of the consumption good lent, in the event that the intermediary monitors entrepreneur \( j \). The contract must be incentive-compatible, as in (3.3), (3.4) and (3.5) (simply replace \( w^d \) by \( w^b \) and \( w \) by \( w_j \)), and we let \( B = \{ w_j: R(w_j) < \hat{R} \} \) and \( B^c = \{ w_j: R(w_j) \geq \hat{R} \} \). As in the previous section, we let \( r \) denote the certain market return. The total return to the intermediary (before compensating depositors) from the \( m \) loan contracts is

\[
\sigma_m - K \sum_{j=1}^{m} \min \{ R(w_j), \hat{R} \}.
\]  

(4.1)

By the strong law of large numbers, we obtain

\[
\lim_{m \to \infty} \frac{1}{mK} \sigma_m = \int_{\hat{R}}^{R(w)} f(w_j) \, dw_j + \frac{R}{\hat{R}} \int_{\hat{R}}^{\infty} f(w_j) \, dw_j.
\]  

(4.2)

Let \( N \) denote the number of borrowers with the intermediary who default in period one. Then \( \sigma_c \) is the utility cost to the intermediary of monitoring these borrowers. Since \( N \) is a binomial random variable with parameters \( (m, \int_{\hat{R}}^{\infty} f(w_j) \, dw_j) \), therefore, by the strong law of large numbers,

\[
\lim_{m \to \infty} \frac{N_c}{mK} = \frac{r}{\hat{R}} \int_{\hat{R}}^{\infty} f(w_j) \, dw_j.
\]  

(4.3)

Therefore, if the following weak inequality holds,

\[
\int_{\hat{R}}^{R(w_j)} f(w_j) \, dw_j + \frac{R}{\hat{R}} \int_{\hat{R}}^{\infty} f(w_j) \, dw_j - \left( \frac{r}{K} \right) \int_{\hat{R}}^{\infty} f(w_j) \, dw_j > r,
\]  

then as the intermediary grows large \( (m \to \infty) \) it can guarantee a certain return of \( r \) to its depositors and attain a level of expected utility of at least \( r \). Given the contract \((R(w), \hat{R})\), a finite-sized intermediary must write contracts with its depositors which involve monitoring and, given the certain market

3The problem of information reliability may go away in repeated games where auditors can establish reputations, and side payments to auditors would likely be illegal. However, the appropriability problem remains.

4In fact, condition (4.4) must hold if intermediaries willingly engage in the contract \((R(w), \hat{R})\) with borrowers.
return \( r \), depositors must be compensated for these monitoring costs by the intermediary. This compensation lowers expected utility for the intermediary, so that, with \( (R(w), \bar{R}) \) given, expected utility is higher for a large (i.e., infinite-sized) intermediary, which depositors need not monitor, than for an intermediary of finite size. As in Diamond (1984), the costs of delegated monitoring go to zero in the limit as the intermediary grows large.

Given that intermediaries will choose to grow large (since this is expected utility-maximizing) for any contract \( (R(w), \bar{R}) \), it remains for us to determine the optimal payment schedule, \( R(w) \), for a large intermediary. Not surprisingly, this is identical to the optimal payment schedule for the direct lending case.

**Proposition 2.** The optimal payment schedule for an intermediary is \( R(w) = w \).

**Proof.** Since depositors need not monitor the intermediary as \( m \to \infty \), and using (4.2) and (4.3), the probability limit as \( m \to \infty \) of the total return per intermediary member (the members are the depositors and the intermediary agent) is given by

\[
V(R(w), \bar{R}) = \int_B R(w) f(w) \, dw + \bar{R} \int_{\mu} f(w) \, dw - \left( c/K \right) \int_{\mu} f(w) \, dw.
\]

The optimal contract maximizes the expected utility of the borrower, subject to \( V(R(w), \bar{R}) \geq r \). That is

\[
\max_{(R(w), \bar{R})} \int_B \left[ w - R(w) \right] f(w) \, dw + \int_{\mu} \left[ w - \bar{R} \right] f(w) \, dw,
\]

subject to

\[
\int_B R(w) f(w) \, dw + \bar{R} \int_{\mu} f(w) \, dw - \left( c/K \right) \int_{\mu} f(w) \, dw \geq r.
\]

By simply replacing \( w \) by \( w \) and \( c \) by \( c/K \) in (3.6), the proof of the proposition is immediate from the proof of Proposition 1. Q.E.D.

As for the case with direct lending, the optimal contract between an intermediary and a borrower is a debt contract and, similarly, the constraint in (4.5) is binding. Therefore, the ‘loan interest rate’, \( \bar{R} \), solves

\[
\max_{\bar{R}} \int_{\mu} w f(w) \, dw,
\]

subject to

\[
\int_{\mu} w f(w) \, dw + \bar{R} (1 - F(\bar{R})) - \left( c/K \right) F(\bar{R}) = r.
\]

Letting \( U(\bar{R}) \) denote the expected utility attained by each depositor and by the intermediary when the loan interest rate is \( \bar{R} \), we have, from (4.6),

\[
U(\bar{R}) = \int_{\mu} w f(w) \, dw + \bar{R} (1 - F(\bar{R})) - \left( c/K \right) F(\bar{R}).
\]

Condition (3.8) then guarantees that \( U(\bar{R}) \) is a concave function on \([0, \bar{w}]\).

We can now define an equilibrium as follows.

**Definition 2.** An equilibrium with intermediation is a loan interest rate \( \bar{R}^* \), a certain market interest rate \( r^* \), and an aggregate loan quantity \( q^* \) which satisfy:

1) \( \bar{R}^* \) solves (4.6).
2) \( q^* = a \int_0^r h(t) \, dt \).
3) Either (a) \( q^* = (1 - \alpha)K \), or (b) \( q^* < (1 - \alpha)K \) and \( 1 - F(\bar{R}^*) - (c/K) f(\bar{R}^*) = 0 \).

As is the case for equilibria with direct lending (Definition 1), there are two types of equilibria: NRA equilibria characterized by (3a) and RA equilibria characterized by (3b). In both types of equilibria, lenders with \( t > r^* \) are indifferent in equilibrium between acting as depositors and acting as intermediaries. Those lenders with \( t > r^* \) invest in their own return projects.

In an RA equilibrium, each entrepreneur offers a loan contract on the market at an interest rate \( \bar{R}^* \), and each intermediary makes loans to entrepreneurs who are chosen at random. However, for each entrepreneur there is a probability \( q^*/(1 - \alpha)K \) of receiving a loan, and, given (3b), if an entrepreneur does not receive a loan, then there is no loan interest rate she can offer which would bid loans away from other entrepreneurs, or which would draw additional lenders into the market to act as intermediaries and depositors. Also, as we will show in the next section, in an RA equilibrium there is no contract that entrepreneurs who do not receive loans could offer directly to lenders that would give these lenders a level of expected utility greater than
r^*, i.e., the intermediation process cannot be circumvented by entrepreneurs who are rationed out of the market.

The fact that lenders who participate in the loan market (lenders for whom \( t \leq r^* \)) are indifferent in equilibrium between acting as intermediaries and acting as depositories is of some interest. In the framework studied by Boyd and Prescott (1985), agents in the model are also free to choose the activities they engage in, though Boyd and Prescott’s framework is more general in this respect than ours. Here, there are assumed attributes of agents which differentiate ‘lenders’ from ‘entrepreneurs’ at the outset.

4.1. NRA equilibrium

Given (4.6) and Definition 2, an equilibrium without rationing is the solution to the following two equations: intermediaries and depositors attain a level of expected utility of \( r \) from (4.6),

\[
\int_0^\bar{R} w_f(w) \, dw + \bar{R}(1 - F(\bar{R})) = r,
\]

and market clearing,

\[
(1 - \alpha)K = \alpha \int_t^\bar{R} h(t) \, dt.
\]

Eqs. (4.8) and (4.9) solve for equilibrium \( \bar{R} \) and \( r \). From (4.6), the following condition also holds in equilibrium:

\[
1 - F(\bar{R}) - (c/K) f(\bar{R}) \geq 0.
\]

4.2. RA equilibrium

From Definition 2, an equilibrium with rationing is the solution to (4.8) and the two following equations: the equilibrium loan rate, \( \bar{R} \), gives the maximum level of expected utility to each intermediary, i.e., it is ‘bank-optimal’, using the terminology of Stiglitz and Weiss (1981)

\[
1 - F(\bar{R}) - (c/K) f(\bar{R}) = 0,
\]

and from condition 2 in Definition 2,

\[
q = \alpha \int_t^\bar{R} h(t) \, dt.
\]

Eqs. (4.8), (4.11) and (4.12) solve for equilibrium \( \bar{R}, r, \) and \( q \).

4.3. Existence and uniqueness of equilibrium

From (4.7) and (4.8) an equilibrium, whether NRA or RA, must satisfy

\[
U(\bar{R}) = r.
\]

Given (3.8), there is some unique \( \bar{R}_{\text{max}} \) which maximizes \( U(\bar{R}) \) on \([0, w] \). Also, let \( \bar{r} \) denote the 'market-clearing' interest rate, i.e., the certain market interest rate which solves

\[
a \int_t^\bar{R} h(t) \, dt = (1 - a)K.
\]

We can then state the following proposition without proof (for brevity).

**Proposition 3.** If \( U(\bar{R}_{\text{max}}) \geq \bar{r} \), then an equilibrium exists and it is unique. Further, if \( t \leq U(\bar{R}_{\text{max}}) < \bar{r} \), then the equilibrium is RA and if \( U(\bar{R}_{\text{max}}) \geq \bar{r} \), then the equilibrium is NRA.

Proposition 3 states that there will be rationing in equilibrium (if the equilibrium exists) if \( \bar{r} > U(\bar{R}_{\text{max}}) \). Since \( U(\bar{R}_{\text{max}}) \) is finite, there exist functions \( h(\cdot) \) and intervals \([t, \bar{R}] \) such that this condition holds [see (4.14)].

4.4. Dominance of direct lending by intermediation

Let \( U_{ij}(\bar{R}) \) denote the counterpart of \( U(\bar{R}) \) for the regime with direct lending, i.e., \( U_{ij}(\bar{R}) \) is the expected utility attained by a direct lender as a function of the loan interest rate. Then from (3.7),

\[
U_{ij}(\bar{R}) = \int_0^{\bar{R}} w_f(w) \, dw + \bar{R}(1 - F(\bar{R})) - cF(\bar{R}).
\]

Then from (4.7), since \( K \geq 2, U(\bar{R}) > U_{ij}(\bar{R}) \) for all \( \bar{R} \in (0, w] \). Suppose then that an equilibrium \((\bar{R}^*, r^*, q^*)\) exists for the regime with direct lending of section 3. Then intermediaries can enter the lending industry, offering entrepreneurs loan contracts at the interest rate \( r^* \), and attain a level of expected utility greater than \( r^* \) for themselves and their depositors, since \( U(\bar{R}^*) > r^* = U_{ij}(\bar{R}^*) \). Note also that if an equilibrium \((\bar{R}^*, r^*, q^*)\) exists for the regime with intermediation, then direct lending cannot occur this, since \( U_{ij}(\bar{R}^*) < r^* = U(\bar{R}^*) \), i.e., in equilibrium lenders prefer to act as intermediary agents or as depositors, rather than as direct lenders. Therefore, intermediation will drive direct lending out of the system.
4.5. Remarks

In our model, as in that of Stiglitz and Weiss (1981), credit rationing may be a characteristic of loan market equilibrium: all entrepreneurs are identical, ex ante, but in equilibrium it may be the case that some receive loans and others do not. A unique feature of our framework is that this phenomenon arises in an intermediate loan market. The costly monitoring of borrowers by lenders creates the possibility of equilibrium credit rationing, while costly monitoring in conjunction with large-scale investment projects implies a role for financial intermediation. The fact that financial intermediation and equilibrium credit rationing are linked in our model is important since (1) most lending in developed economies occurs through intermediaries and (2) credit rationing is usually associated in the literature with intermediated lending.

Here, as in papers by Boyd and Prescott (1985) and Diamond (1984), financial intermediation dominates direct lending as a means for financing investment projects under asymmetric information. In our model and those of Boyd and Prescott and Diamond, intermediation produces information more efficiently and does this through diversification, in spite of universal risk neutrality.

The financial intermediaries in our model exhibit several important features of intermediaries as we know them. They issue securities with return characteristics which are different from those of the assets they hold, they manage a diversified portfolio, their assets are debt claims, and they process information. Intermediaries in this framework also may ration credit in equilibrium, though credit rationing has much the same status as involuntary unemployment in terms of its acceptance as an empirical fact.

5. Comparative statics

In this section we examine the responses of equilibrium \( \bar{R} \), \( r \), and \( q \) to changes in parameters and distribution functions. These responses are quantitatively different and frequently qualitatively different, depending on whether the equilibrium is RA or NRA.

We carry out three experiments. Experiment 1 is a shift in the schedule of alternative rates of return faced by intermediaries, interpreted as a downward shift in the supply of funds faced by intermediaries. That is, the function \( h(\cdot) \) shifts to \( h^*(\cdot) \), where \( h^*(t) = h(t - \delta) \), with \( \delta > 0 \). Experiment 2 is a change in \( c \), the cost of monitoring, and experiment 3 is a mean preserving spread in the distribution of project returns, \( f(\cdot) \). That is, in experiment 3 we change \( f(\cdot) \) by shifting probability mass from around equilibrium \( \bar{R} \) to the tails of the

It is no accident that Stiglitz and Weiss (1981) refer to their lending institutions as 'banks', though Stiglitz-Weiss 'banks' have few bank characteristics which are not imposed.
5.2. RA equilibrium

An RA equilibrium is the solution to eqs. (4.8), (4.11) and (4.12). Here, in contrast to the NRA equilibrium, changes in the underlying cost parameters and distributions will in general change \( r \), and this will change \( q \), the aggregate quantity of loans, and the quantity of rationing. Since an increase in \( r \) leads to an increase in \( q \) (except for experiment 1), we need to determine only the sign of the change in \( r \) to determine the qualitative effect on \( q \).

We let \( \nabla_2 \) denote the following quantity:

\[
\nabla_2 = f(\bar{R}) + (c/K)f'(\bar{R})
\]

Given (3.8), we have \( \nabla_2 > 0 \).

Totally differentiating eqs. (4.8) and (4.11) and solving, we obtain:

**Experiment 1.** \( d\bar{R}/d\varepsilon = dr/d\varepsilon = 0, dq/d\varepsilon < 0 \).

**Experiment 2.** \( d\bar{R}/dc = -f(\bar{R})/K\nabla_2 < 0, \quad dr/dc = -F(\bar{R})/K < 0, \quad dq/dc < 0 \).

**Experiment 3.** \( d\bar{R}/d\delta \bigg|_{\delta = 0} = -cg(\bar{R})/K\nabla_2 > 0, dr/d\delta \bigg|_{\delta = 0} = f^R_wg(w)dw < 0, dq/d\delta \bigg|_{\delta = 0} < 0 \).

The quantitative effects on \( \bar{R} \), \( r \) and \( q \) of each experiment always differ, depending on whether the equilibrium is RA or NRA. Also, except for the effect on \( \bar{R} \) in experiment 3, variables either move in different directions in the NRA and RA equilibria or remain unchanged in one type of equilibrium while increasing or decreasing in the other. In addition, note the following:

1. In the RA equilibrium, all experiments have an effect on the aggregate quantity of loans, \( q \), while these effects are absent in the NRA equilibrium.\(^{10}\) Note also, in this respect, that experiment 1 has no effect on interest rates in the RA equilibrium. This result is consistent with the thrust of the availability doctrine [see Roosa (1951)], i.e., monetary policy can have real effects without affecting interest rates in lending markets. For there to be any real effects, of course, monetary policy must change some real interest rate(s), and just how this might occur would have to be worked out by embedding our model in a general equilibrium framework.

\(^{10}\)In general, there would be effects on the quantity of loans in the NRA equilibrium if the demand for loans were not inelastic, what is important here is that there are effects on the amount of rationing in the RA equilibrium which are absent in the NRA equilibrium.

2. For experiment 1, in the NRA equilibrium an increase in \( r \) not only increases \( \bar{R} \), but also increases the difference between \( R \) and \( r \). This result was also obtained in Williamson (1983, 1984b) and is consistent with the stylized fact that interest rate differentials increase with an increase in all interest rates.

3. With an increase in the riskiness of entrepreneurs’ investment projects in experiment 3, the loan interest rate, \( \bar{R} \), increases, i.e., there is a risk premium effect in spite of the fact that all agents are risk-neutral. This occurs due to a corresponding increase in the probability of default for entrepreneurs which increases the expected cost of monitoring.

From Proposition 3, we note that anything that would increase \( R \) or decrease \( U(\bar{R}_{\text{max}}) \) would make an equilibrium with credit rationing more likely. First, given (4.14), experiment 1 will reduce \( R \). Next, to determine what will bring about decreases in \( U(\bar{R}_{\text{max}}) \), note that in an RA equilibrium \( U(\bar{R}_{\text{max}}) = r \). Therefore, experiments 2 and 3 will reduce \( U(\bar{R}_{\text{max}}) \). We then conclude that increases in alternative rates of return, in monitoring costs, and in project riskiness all increase the likelihood of equilibrium credit rationing.

6. Summary and conclusion

A shortcoming of previous studies in the credit rationing literature [for example, Jaffe and Russell (1976), Keeton (1979) and Stiglitz and Weiss (1981)] is that the lending institutions in these models have few of the features one would associate with real-world intermediaries (other than what is assumed), in spite of the fact that these analyses are often clearly intended to apply to intermediated markets. The main purpose of this paper has been to demonstrate that equilibrium credit rationing can occur in an environment where financial intermediation is motivated from first principles. In fact, in the model considered, intermediation and credit rationing are related phenomena, in that the same set of assumptions can produce both.

There are two types of equilibria in our model, those with credit rationing and those without. Both types of equilibria are possible either with direct lending (if intermediation is prohibited) or in a regime with financial intermediation. Credit may be rationed in the sense of Stiglitz and Weiss (1981), i.e., that all entrepreneurs (potential borrowers) are identical, ex ante, but it may be the case that some receive loans and others do not. In contrast to Stiglitz and Weiss (1981), this does not occur due to moral hazard or adverse selection, though informational asymmetries are crucial. Instead, the costly monitoring of borrowers by lending agents (either intermediaries or direct lenders) implies, given risk neutrality, that debt contracts are optimal. There is therefore an asymmetry in the payoff functions of borrowers and lenders, which creates the possibility of equilibrium credit rationing.

Debt contracts are also derived as the optimal arrangement between borrowers and lenders in Diamond (1984) and Gale and Hellwig (1985), though
the costs of 'bankruptcy' in Diamond's paper are non-pecuniary costs to the borrower rather than monitoring costs, as in our model. Debt contracts are an important element in generating equilibrium credit rationing in Stiglitz and Weiss (1981), as they are here. However, Stiglitz and Weiss simply assumed that the contracting arrangement took this form.

Financial intermediation dominates direct lending in our model as a result of costly monitoring and large scale investment projects. As in Diamond (1984), intermediation performs a 'delegated monitoring' role, and intermediaries are single agents. However, in our model we do not distinguish separate intermediary agents at the outset as Diamond does, but instead allow agents in the model to freely choose activities given their endowments and preferences. Such an approach is also taken in Boyd and Prescott (1985), though in their framework intermediaries are multi-agent coalitions. Financial intermediaries in our model share several of the important features of intermediaries as we know them; they issue securities which have payoff characteristics which are different from those of the securities they hold, they write debt contracts with borrowers, they hold diversified portfolios, and they process information.

It was shown that an equilibrium with intermediation, if it exists, is unique and that this equilibrium will be one of two types: either credit is rationed or it is not. An equilibrium can be simply characterized, and it is then relatively straightforward to derive implications concerning the effects on observable variables (market interest rates and the quantity of lending) of changes in the parameters of preferences and technology. Similar implications are not a part of any of the other financial intermediation studies cited here. These implications, which are broadly consistent with the results in Williamson (1984b) for direct lending, follow from the comparative statics experiments in section 5 of the paper and can be summarized as follows.

1) The responses of the endogenous variables are always quantitatively different for the two types of equilibria and are frequently qualitatively different. 2) Quantity effects which are absent in an equilibrium without rationing are a feature of rationing equilibria. 3) There are risk premium effects on the loan interest rate in spite of universal risk neutrality. 4) An increase in all interest rates is consistent with an increase in interest rate differentials. 5) Changes in alternative rates of a return can produce changes in the quantity of lending without interest rate effects.

We note in conclusion that it is not clear, in spite of the fact that some of the results might be interpreted as being consistent with the availability doctrine, that such an interpretation would provide a role for monetary policy as envisioned by proponents of this doctrine [see Roosa (1951)]. To draw normative conclusions for monetary policy, our model would have to be embedded in a more fully-specified dynamic general equilibrium framework.

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