Currency elasticity and banking panics:
thory and evidence

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Abstract. Existing models of banking panics contain no role for monetary factors and fail to explain why some banking systems experienced panics while others did not. A monetary model is constructed, where seasonal variations in the demand for liquidity and credit play a critical role in generating banking panics. These panics occur when there are restrictions on the issue of currency by private banks, but they do not occur if banks are unrestricted. Empirical evidence from Canada and the United States for the period 1880–1910 is largely consistent with the predictions of the model.

Régimes monétaires plus ou moins élastiques et paniques bancaires: théorie et résultats. Les modèles de paniques bancaires qu’on possède ne donnent aucun rôle aux facteurs monétaires et n’expliquent pas pourquoi certains systèmes bancaires font l’expérience de mouvements de panique alors que ce n’est pas le cas pour d’autres. Les auteurs construisent un modèle monétaire où les variations saisonnières dans la demande de liquidité et de crédit jouent un rôle important dans la genèse des paniques bancaires. Ces paniques se produisent quand il y a des restrictions sur l’émission de monnaie par les banques privées mais ne se produisent pas si on n’impose pas cette restriction aux banques. Les résultats empiriques pour la période 1880–1910 au Canada et aux États-Unis confirment largement les prédictions du modèle.

We have benefited from helpful discussions with Michael Bordo, Charlie Calomiris, Dick Highfield, Jeff Miron, Ed Prescott, Neil Quigley, Angela Redish, Arthur Rolnick, Richard Sylla, and Warren Weber. We would also like to express appreciation for the constructive comments provided by two anonymous referees, as well as participants at the 1991 Canadian Economics Association meetings, the 1991 Federal Reserve Bank of Minneapolis Conference on Monetary Theory and Financial Intermediation, and seminars at the Federal Reserve Bank of Atlanta, the Federal Reserve Bank of Dallas, the Federal Reserve Bank of St Louis, Fordham University, Queen’s University, Rice University, the University of California at Los Angeles, the University of California at San Diego, the University of Houston, the University of Illinois, the University of Miami, the University of Texas at Austin, and the University of Western Ontario. We alone are responsible for any errors or omissions. Finally, the financial support of the Lynde and Harry Bradley Foundation and the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged, as is the research assistance of Pantelis Andreaou, Chris Konishi, and Jing Xu.
1. INTRODUCTION

The original developments in Bryant (1980) and Diamond and Dybvig (1983) spawned a large literature on bank panics and suspensions of convertibility. The class of Diamond and Dybvig type models has added to our understanding of bank liquidity provisions and has given a clear view of the potential difficulties caused by a natural mismatch of maturities between bank assets and liabilities. These models also formalize a number of contemporary explanations for the occurrence of panics/suspensions, including self-fulfilling prophecies (Diamond and Dybvig 1983), excessive competition for deposits (Smith 1984), and adverse inferences caused by heavy ('fundamental') withdrawal demand (Chari and Jagannathan 1988).

While Diamond-Dybvig-type models explain banking panics as inherent to the structure of deposit contracts, other work has focused on the role of the regulatory environment in fostering or deterring panics. Historically, the record of panics has been quite different in banking systems operating under different regulatory regimes. For example, the dramatically different panic experiences (or lack thereof) in the United States and Canada is well documented (Breckenridge 1910; Johnson 1910; Myers 1931; Williamson 1989; Haubrich 1990), and a number of authors have pursued different explanations of why Canada was able to avoid the frequent panics observed in the United States. For instance, in contrast to the system in effect in the United States, Canadian banks were able to branch freely. White (1984), Williamson (1989), and Smith (1991) explore this as a potential explanation for the relative stability of the Canadian banking system. Somewhat relatedly, the Canadian system appears to have been conducive to a greater degree of cooperation/collusion among individual banks than was the U.S. system. Some contributions (Smith 1984; Williamson 1989; Calomiris 1989a, b; Calomiris and Gorton 1990) identify this as a possible stabilizing feature.

Diamond-Dybvig-type models, and other similar constructs, consider purely 'real' economies – ones that lack a role for currency. One objective of this paper is to develop a model in which the mechanisms for money creation and the provision of liquidity by banks are related. There are at least four important reasons for expanding the scope of banking panic models by introducing monetary considerations. First, operational definitions of panics involve currency in a central way. For example, the defining characteristics of a banking panic, according to Noyes (1909, 186–7), include the suspension of cash payments by banks to depositors, the depletion of cash reserves at banks, the emergence of a currency premium, and the adoption of 'emergency expedients ... to provide the necessary medium of exchange for ordinary business.' Second, most contemporary proposals for reforming

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1 The closest event to a U.S.-style panic in Canadian history appears to have occurred in 1837–8 during a period of political unrest. For an account see Redish (1983).
2 Other important theoretical contributions include Gorton (1985), Bhattacharya and Gale (1987), Bental, Eckstein, and Peled (1989), Calomiris and Kahn (1990), and Donaldson (1989). All of them present real economies. For a survey of some of this literature, see Bhattacharya and Thakor (1993).
the U.S. banking and monetary systems reflected a belief regarding an intimate relationship between the monetary systems and panics. For example, monetary and banking reforms were proposed in the ‘Baltimore Plan’ of 1894 and by the Indianapolis Monetary Commission of 1900 (West 1977, 43–5). Some proposals argued that the United States should have a monetary system like that of Canada, which had a ‘truly elastic currency’ (Laughlin 1912, 130). The desire to reform the system for issuing currency and/or banknotes was also reflected in the Aldrich/Vreeland Act and the Federal Reserve Act. Third, a number of contemporary observers (including Laughlin 1912, 312) argued that the system of banknote issue was more important than branching restrictions in explaining the apparent stability of the Canadian banking system. Fourth, a great deal of our empirical knowledge about banking panics, such as that summarized in Friedman and Schwartz (1963) and Miron (1986), concerns the behaviour of money, credit, the reserve-deposit and currency-deposit ratios, and nominal interest rates during panic episodes. Clearly an understanding of these observations requires that monetary factors be integrated into existing banking models.

In constructing a model of panics that attaches importance to monetary factors, we also attempt to capture some contemporary explanations of banking panics in which seasonal variations in the demand for credit and liquidity play a role. For instance, Andrew (1908a) and Sprague (1910) (also see Goodhart 1969 for a concise summary) argued that periods of ‘crop-moving’ – when credit demand and liquidity needs tended to be greatest – placed significant seasonal strains on the banking system. At these times relatively small incremental shocks to the demand for currency could push the banking system into a panic. While this view was widely held by contemporary observers (and subsequent students, such as Goodhart), it has found little reflection in existing models of panics (important exceptions include Miron 1986 and Chari 1989). It has also come under some historical (Patterson 1913) as well as more recent (Calomiris and Gorton 1990) attack. Our analysis provides some new perspectives on this discussion, reconciling the Andrew and Sprague accounts with the criticisms of Patterson and Calomiris and Gorton.

In the sequel, we undertake both a theoretical and an empirical analysis of these issues. We develop a model where banks exist to insure agents against random needs for liquidity (as in Diamond and Dybvig 1983) and to intermediate between lenders and borrowers. We consider a pure exchange economy with overlapping generations of two-period-lived consumers. Within each generation some agents are lenders and some are borrowers. Agents are assigned to either of two locations. After exchange and consumption occur in the first period of life, a randomly selected fraction of lenders is forced to relocate. As in Townsend (1987), Mitsui and Watanabe (1989), or Hornstein and Krussell (1990), only paper liabilities can be transported between locations, and limited communication prevents claims on specific agents from being traded across locations. This generates a transactions role for currency. Moreover, the possibility of relocation takes the place of ‘preference shocks’ in Diamond and Dybvig (1983); in the event of relocation an agent needs liquidity. Banks, in addition to making loans, hold reserves, issue notes if permitted, and write deposit
contracts insuring lenders against the possibility of relocation. Thus the monetary and banking systems are intimately related.

In addition to aggregate (as well as individual) randomness associated with relocations, there are two sources of deterministic seasonal variation in the model. As in Sargent and Wallace (1982), which in many ways our model resembles, there is a seasonal variation in credit demand. In addition, we introduce seasonal fluctuations in ‘liquidity preference,’ captured here by deterministic seasonal variation in the probability distributions governing relocation. In the model, high credit demand and greater liquidity preference coincide seasonally, as Laughlin (1912, 314–15) argues was true historically. This is an attempt to capture the seasonal impacts on historical financial markets associated with the needs of ‘crop-moving’ in a highly agricultural economy.

Having constructed the model, we then consider two banking regimes. In the first, banks can issue notes (which are default-free claims to currency) against general assets. In the second, banks are prohibited from issuing notes, so that currency is entirely a liability of the government. We are interested in contrasting the operating characteristics of these two systems, the first of which we refer to as an ‘elastic’ and the second as an ‘inelastic’ currency regime. These regimes, as well as the model itself, are set up with the data in mind; in particular, they are designed to capture certain important features of the banking environments in Canada and the United States during the period 1880–1910. This period contains that of the National Banking System in the United States (1863–1913), during which all U.S. currency was either a direct liability of the government or in the form of national banknotes backed by government bonds (being, therefore, de facto liabilities of the government). Moreover, the mechanics of note issue severely limited currency elasticity, so that our inelastic currency regime captures some essential features of the U.S. system. In Canada during this period, chartered banks were free (except for limitations imposed by capital requirements and denominational restrictions) to issue notes against general assets. Institutional arrangements served to make these notes essentially default free (Johnson 1910; Breckenridge 1910; Williamson 1989), so that our elastic currency regime corresponds to the Canadian system.

In the elastic currency regime, banks can use note issues to accommodate completely withdrawals arising from relocations. In this situation there is a Pareto optimal equilibrium, bank panics do not occur, and the nominal interest rate is always zero. Deterministic fluctuations and random disturbances to liquidity demand are concentrated in fluctuations in the stock of inside money (inside money is ‘elastic’) rather than nominal interest rates. This regime thus has some features of the ‘real bills’ regime studied by Sargent and Wallace (1982). Our model, however, need not have the same implications for price-level variability as that of Sargent and Wallace.

Under the inelastic currency regime the nominal interest rate is positive and varies seasonally, being highest in periods of greatest financial market pressure. In addition, banking panics occur with positive probability, where a panic corresponds to a situation of complete exhaustion of bank reserves and suspension of cash
payments. Since other bank assets are illiquid, this event precludes depositors from being fully insured. In contrast to Diamond and Dybvig, however, our model has no private information or sequential service constraint, and panics are the result of fundamentals. In the event of a panic, payments are made optimally to agents making withdrawals. The probability of a panic is highest in periods of greatest seasonal ‘pressure,’ and in the event of a panic a currency premium can be observed. Moreover, a panic will correspond to the attainment of a critical value by the currency-deposit ratio, which is exactly how panics are described by Friedman and Schwartz (1963, 56, 109, 122, 158, 333). Finally, the inelastic currency regime does not support a Pareto optimal equilibrium.

In confronting the model with empirical observations, we undertake an analysis of monthly data on loans, deposits, reserves, note issues, and nominal interest rates for the United States and Canada over the period 1880–1910. The data are decomposed into trend, seasonal, and cyclical components, and the seasonal and cyclical components are contrasted with the implications of the model. As the model predicts, the Canadian data display greater seasonal variation of banknotes in circulation than those of the United States, and this greater degree of currency elasticity permits less seasonal variability in loans, deposits, and nominal interest rates. In Canada loans expand in the autumn to accommodate crop-moving needs; whereas in the United States loans contract at this time and nominal interest rates rise. With respect to cyclical variability, Canadian banknote issues rise markedly (but not exclusively) during panic periods in the United States. This permits Canadian banks to avoid the loss of cash reserves at these times, in marked contrast to their U.S. counterparts.

The remainder of the paper proceeds as follows. In section II the model is constructed and an equilibrium for the elastic currency (unrestricted note issue) regime is studied. The implications of an inelastic currency are analysed in section III. In section IV a comparison of the two regimes is presented. In section V there are comments on the Andrew-Sprague-Goodhart ‘seasonal plus shock’ theory of panics, discussing how the analysis permits this view to be reconciled with the criticisms of Patterson and of Calomiris and Gorton. Concluding comments are offered in section VI.

II. THE MODEL

1. Environment

We consider an overlapping generations economy, with time indexed by \( t = 1, 2, \ldots \). In each period, a continuum of two-period-lived agents with unit mass is born at each of two locations. Half of these agents are ‘lenders’ and the remaining half are ‘borrowers.’ All agents have preferences given by

\[
u(c_1, c_2) = \ln c_1 + \beta \ln c_2,\]

where \( c_j \in \mathbb{R}_+ \) denotes consumption in period \( j \) of life. Lenders have an endowment \( x > 0 \) of the non-storable consumption good when young and no endowment when old. Borrowers have no endowment when young, and a borrower born at date \( t \) has an endowment \( y_t \) when old, where \( y_t = y_o \) for \( t \) odd and \( y_t = y_e \) for \( t \) even, with \( y_o > y_e > 0 \). Also, we assume that \((\beta x)^2 > y_o y_e\), which implies that this is
a ‘Samuelson case’ economy (Gale 1973). At \( t = 1 \) there is a continuum of old agents with unit mass in each location. Each of these agents is endowed with \( M \) units of fiat currency, where \( M > 0 \), and there are no subsequent injections or withdrawals of currency.\(^3\)

As is typical in models where movement between locations is taken seriously, the timing of events within a period and the pattern of meetings between agents is of considerable importance. At the beginning of each period agents in each location receive their endowment. At this time young lenders are able to deposit resources with a bank, whose behaviour is described below, and they can also trade with old agents. After making these transactions, young lenders consume, and are then unable to make contact with other agents until they learn whether or not they are to be relocated. (In particular, young lenders and young borrowers never meet.) Young borrowers contact the bank next, taking loans in a manner we describe below. After loans have been made, young lenders learn whether or not they are to be relocated. At time \( t \) a random fraction \( \pi_t \) of young lenders, denoted ‘movers,’ must relocate, with \( \pi_t \) being the same in each location (to preserve symmetry). Agents who are not relocated are denoted ‘non-movers.’ After learning their status, movers are able to contact their bank, withdrawing their deposits plus any promised interest. Since all time \( t \) resources have been consumed, these agents receive claims to future consumption in the form of either banknotes or currency, following which relocation occurs. Then at \( t+1 \) these liabilities can be used to purchase goods when old agents contact young lenders and/or a bank in their new location. Consumption goods cannot be transported between locations, but banks can trade banknotes and currency between locations at the beginning of the period. Direct claims on agents (other than banks) in one location are counterfeitable in the other location and therefore have no value there. In particular, limited communication prevents the quality of cheques written by movers from the other location or of claims on borrowers in the other location from being verified. Banknotes and currency cannot be counterfeited, however, and our assumptions will imply that these two assets are perfect substitutes in equilibrium, from the point of view of movers.

The relocation probability \( \pi_t \) itself is assumed to be a random variable, which is drawn from the distribution \( F_t(\cdot) \). We assume that \( F_t(\cdot) = F_o(\cdot) \) for \( t \) odd and \( F_t(\cdot) = F_e(\cdot) \) for \( t \) even, with \( F_o \) FSD \( F_e \), that is, \( F_o \) first-order dominates \( F_e \), or \( F_o(\pi) \leq F_e(\pi), \forall \pi \). The distribution function \( F_t(\cdot) \) has the associated continuously differentiable density function \( f_t(\cdot); t = o, e \). Finally, we assume that a young lender’s status as a mover or non-mover is publicly observable and that bank payments can be made contingent on this status.

The assumptions that \( F_o \) FSD \( F_e \) and that \( y_o \geq y_e \) are meant to reflect the seasonal pressures on the banking system associated with crop moving. \( F_o \) FSD \( F_e \) implies that, probabilistically speaking, the demand for currency is high in odd periods (‘autumn’), while \( y_o \leq y_e \) implies that loan demand is high at the same

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\(^3\) As will be apparent, the exact specification of endowment patterns and population proportions is without loss of generality. Also, we note that the environment, to this point, is a special case of that described in Sargent and Wallace (1982).
time. These assumptions coincide with standard historical accounts of the seasonal factors affecting the U.S. and Canadian banking systems.

a. Remarks
The basic structure of the model is similar to that of Sargent and Wallace (1982), with two ‘seasons’ and seasonal variation in credit demand. We have added the feature that individuals have idiosyncratic and random needs for liquid assets (assets that can be used in other locations), as in Diamond and Dybvig (1983). The nature of interlocation exchange introduces a role for a medium of exchange that is familiar from Townsend (1987), Mitsui and Watanabe (1989), and Hornstein and Krusell (1990).

Relative to Sargent and Wallace (1982), we have also introduced seasonal factors affecting the demand for media of exchange (through the seasonal variations in \( F_t(\cdot) \)). Our interest is in part to see how different banking regimes respond to seasonal factors affecting the demand for credit and liquidity. Andrew, Sprague, and Goodhart, for instance, argue that the role of seasonal factors is intimately connected with the occurrence (or in Canada non-occurrence) of panics. We have assumed that credit and liquidity demands (in a probabilistic sense) tend to be high simultaneously. Laughlin (1912, 314–15) argues that this is a historically accurate description of the pressures placed on financial markets by crop-moving in the agricultural economies of the United States and Canada.

An important feature of the model is that interlocation exchange must be accomplished with currency or banknotes. In the model this is a consequence of limited communication between locations. This captures an important historical phenomenon; for as Sprague (1910, 75) asserted, ‘in making payments at a distance local substitutes for money will not serve,’ where ‘local substitutes’ includes cheques or other private liabilities.4

2. Behaviour of agents
We assume that there is a fixed, finite number of infinitely lived banks in each location.5 These banks take deposits, hold reserves, and make loans. In addition, banks announce pay-off schedules at \( t \), which specify gross real returns to depositors contingent on type (mover versus non-mover) per unit deposited. We let \( r_t^m(\pi_t)[r_t(\pi_t)] \) denote the one-period return for movers (non-movers), contingent on \( \pi_t \). Each bank announces such a schedule, taking the announcements of other banks as given. Once return schedules are announced each bank simply accepts all deposits offered and makes loans charging the competitively determined gross (real) loan rate \( R_t \).6

4 For corroboration or even stronger assertions on this point see Stevens (1894, 140), Laughlin (1912, 323), Goodhart (1969, 6), or Rich (1989, 150), who also makes clear that the same situation existed in Canada.
5 Each bank takes deposits and makes loans only in its own location.
6 Note that \( R_t \) is not contingent on \( \pi_t \). When banks are note issuing, this lack of state contingency is innocuous. When banks are not note issuing, it is not, but lack of state contingency could be motivated by observation that borrowers are physically separated from lenders when \( \pi_t \) is realized. Borrowers therefore do not observe \( \pi_t \), making it natural not to condition repayments on it.
Having observed announced repayment schedules at $t$, each lender chooses a savings level (or deposit) $d_t$ and a bank. As in Diamond and Dybvig (1983), all savings will be deposited, so $d_t$ is chosen to maximize the lenders expected utility, which is

$$
\ln (x - d_t) + \beta \int_0^1 \pi \ln [r_t^m(\pi)d_t]f_t(\pi)d\pi + \beta \int_0^1 (1 - \pi) \ln [r_t(\pi)d_t]f_t(\pi)d\pi.
$$

The solution to the problem sets $d_t = \beta x/(1 + \beta)$, and each lender chooses the bank whose return schedules maximize this expression.

Borrowers observe the competitively determined gross loan rate $R_t$ at $t$ and choose a loan quantity $l_t$ to maximize $\ln l_t + \beta \ln (y_t - R_t l_t)$. The solution to this sets $l_t = y_t(1 + \beta)R_t$.

a. The bank’s problem

Banks annunciate return schedules, take deposits, make loans, hold currency and reserves, and (possibly) issue notes. Thus, banks face a non-trivial portfolio allocation problem, which affects what return schedules can be announced. We now describe the choices open to banks.

Consider a bank that in period $t$ has a positive measure of depositories, with $d_t$ denoting the real value of deposits per depositor. Against deposits, the bank holds per depository cash reserves with a real value of $z_t$ and makes loans (per depositor) with a real value of $d_t - z_t$. Let $\gamma_t \equiv z_t/d_t$ be the bank’s reserve-deposit ratio. Loans earn the one-period gross return $R_t$, and reserves earn the one-period real gross return $p_{t+1}/p_t$, where $p_t$ is the time $t$ inverse price level. The bank takes $R_t$ and $p_{t+1}/p_t$ as given.

After $y_t$ is chosen and loans are made at $t$, $\pi_t$ is realized. Then the bank faces real per depositor withdrawal demand equal to $d_t\pi_t r_t^m(\pi_t)p_t/p_{t+1}$.

This payment is made by the bank in the form of currency or notes; since borrowers have no resources and have already consumed at the end of period $t$, there is no ability to liquidate loans. Movers then take the currency or notes to their new location, where they are used to make purchases at $t + 1$.

Let $\alpha_t(\pi_t)$ denote the fraction of its cash reserves that the bank pays out at $t$ (as a function of $\pi_t$), and let $b_t(\pi_t)$ be the real value of notes (per depositor) issued by the bank at $t$. (Banknotes are claims to currency that are redeemable at any date in the future.) Then payments to movers at $t$ must satisfy the constraint

$$
\pi_t r_t^m(\pi_t)p_t/p_{t+1} \leq \alpha_t(\pi_t)\gamma_t + b_t(\pi_t)/d_t,
$$

or equivalently,

$$
\pi_t r_t^m(\pi_t) \leq \alpha_t(\pi_t)\gamma_t p_{t+1}/p_t + b_t(\pi_t)p_{t+1}/d_t p_t.
$$

The presence of the term $p_t/p_{t+1}$ reflects the fact that the bank gives currency or notes to movers at $t$. These earn the real rate of return $p_{t+1}/p_t$ between $t$ and $t + 1$. Thus, a payment of $r_t^m(\pi_t)p_t/p_{t+1}$ to movers at $t$ results in a perceived return to the mover of $r_t^m(\pi_t)$.
Payments to non-movers at \( t + 1 \) are \((1 - \pi_t)d_t r_t(\pi_t)\). They cannot exceed the value of the bank’s remaining assets, which are its remaining reserves plus its loan repayments. Also, if the bank retires (or redeems) all notes issued at \( t \), this absorbs resources equal to the real value of note issues times the gross return on notes; that is, \( b_t(\pi_t)p_{t+1}/p_t \) per depositor. Therefore, assuming complete redemption of time \( t \) note issues at \( t + 1 \),

\[
d_t(1 - \pi_t)r_t(\pi_t) \leq d_t \gamma_t [1 - \alpha_t(\pi_t)] p_{t+1}/p_t + d_t (1 - \gamma_t) R_t - b_t(\pi_t)p_{t+1}/p_t. \tag{2}
\]

Finally, we impose the constraints \( 0 \leq \gamma_t \leq 1 \), \( 0 \leq \alpha_t(\pi_t) \leq 1 \), and \( 0 \leq b_t(\pi_t) \).

3. Nash equilibrium return schedules: note issue permitted

We now describe Nash equilibrium contract announcements by banks under the assumption that note issue is unrestricted. Clearly, and Nash equilibrium has banks earning zero profits; therefore (1) and (2) hold with equality in equilibrium. In addition, competition among banks for deposits implies that Nash equilibrium contract announcements must maximize the expected utility of depositors, taking ‘deposit demand schedules’ as given. Since \( d_t = \beta x/(1 + \beta) \), \( r_t^{m}(\pi_t) \), and \( r_t(\pi_t) \) must (in equilibrium) be chosen to maximize the expression

\[
\ln [x/(1 + \beta)] + \beta \int_0^1 \pi \ln [r_t^{m}(\pi_t) \beta x/(1 + \beta)] f_t(\pi) d\pi
\]

\[
+ \beta \int_0^1 (1 - \pi) \ln [r_t(\pi) \beta x/(1 + \beta)] f_t(\pi) d\pi,
\]

subject to (1), (2), and the standard non-negativity constraints, with \( R_t \) and \( p_{t+1}/p_t \) taken as given.

The solution to this problem satisfies (1) and (2) with equality and sets \( r_t^{m}(\pi_t) = r_t(\pi_t) \forall \pi_t \). Therefore,

\[
r_t^{m}(\pi_t) = r_t(\pi_t) = \gamma_t p_{t+1}/p_t + (1 - \gamma_t) R_t. \tag{3}
\]

In addition, \( \gamma_t = 0 \) (1) if \( R_t > (p_{t+1}/p_t) \). Thus, an equilibrium with loans and valued fiat currency has \( R_t = p_{t+1}/p_t \). This is equivalent to loans bearing a zero nominal interest rate. In this situation \( y_t \) is indeterminate, from the standpoint of any individual bank. It is also the case that the schedules \( \alpha_t(\pi_t) \) and \( b_t(\pi_t) \) are not uniquely determined. From (1) and (3), however,

\[
\alpha_t(\pi_t) \gamma_t + b_t(\pi_t)/d_t = \pi_t. \tag{4}
\]

8 We would get the same results in what follows if notes issued in period \( t \) were assumed to be claims to currency in period \( t + 1 \) only, with zero redemption value thereafter. Here, we follow the route of conjecturing an equilibrium where all notes are redeemed in the period after their issue, and then we show that all equilibrium conditions are satisfied.

9 The assumption of logarithmic utility implies that the solution to this problem would be unaffected if banks did not take depositor behavior as given (i.e., if they viewed themselves as being able to choose \( d_t \) as well as return schedules). The same remark applies in section III.1, when banks are not note issuing.
Here, \( \alpha_i(\pi_i)\gamma_i d_t \) is the amount of its cash reserves the bank pays to its depositors at \( t \), and \( b_i(\pi_i) \) is the bank’s note issue per depositor (in real terms). Thus, the quantity of circulating media of exchange transferred from banks to depositors is determined at each date.\(^1\)

4. General equilibrium: note issue permitted

We now describe the determination of a full general equilibrium with valued fiat currency (under the assumption that all notes are redeemed in one period). Then \( R_t = p_{t+1}/p_t, \forall t. \)\(^2\) Moreover, the net per capita savings of each young generation must equal the per capita supply of real balances in each period so that, \( \forall t, \)

\[
\frac{\beta x}{1 + \beta} - y_i/(1 + \beta)R_t = p_t M.
\] (5)

Finally, all real balances are held as cash reserves by banks, so that in equilibrium,

\[
\gamma_i d_t = \frac{\gamma_i \beta x}{1 + \beta} = p_t M .
\] (6)

As in Sargent and Wallace (1982), we confine attention to periodic equilibria where \( R_t = R_t, p_t = p - i \), and \( y_t = y_i \), where \( i = o \) for \( t \) and \( i = e \) for \( t \) even. Substituting \( R_t = p_{t+1}/p_t \) into (5) and using (5) for \( t = o, e \) gives

\[
\frac{\beta x - (y_o/R_o)}{\beta x - (y_e/R_e)} = p_o/p_e = R_e = 1/R_o .
\] (7)

Solving (7),

\[
1/R_o = R_e = (\beta x + y_e)/ (\beta x + y_o) < 1 .
\] (8)

Then, \( p_o \) and \( p_e \) can be obtained from (5) and (8), and \( \gamma_o \) and \( \gamma_e \) can be obtained from (6). It is easy to verify that \( p_e > p_o > 0 \), and \( \gamma_e > \gamma_o \). Thus real interest rates and the price level \( (1/p_t) \) are highest when credit demand is greatest. Notice that liquidity shocks have no effect on any equilibrium values; in particular, the probability distributions \( F_t(\cdot) \) themselves are irrelevant for all equilibrium quantities. This is because when banks are note issuing, it is possible for them to insure perfectly against all relocation risk.

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\(^1\) Our discussion presumes that all banknote issues are redeemed after one period. Champ, Smith, and Williamson (1994) demonstrates that this is, indeed, an equilibrium behavior. As a practical matter, the Canadian banking system seems to have achieved this kind of prompt note redemption. See Johnson (1910, 23) for elaboration on this point.

\(^2\) Since \( R_t = p_{t+1}/p_t \), currency is not dominated in rate of return. Thus all lenders are indifferent between holding currency and making bank deposits. It is easy to verify that, if lenders hold some currency directly, this affects no aspects of an equilibrium except the aggregate reserve-deposit ratio. In particular, if a fraction \( \mu_t \) of the aggregate savings of lenders is held directly in the form of currency, the equilibrium reserve-deposit ratio satisfies \( [\mu_t + \gamma_t(1 - \mu_t)]\beta x/(1 + \beta) = p_t M \), and \( p_t \) as given in the text. In short, banks simply reduce reserves by the amount of currency that lenders hold directly.
It is also the case that there are no panics in this economy. A panic here (see section III.1) occurs when banks are unable to provide sufficient media of exchange to allow \( r^m_t(\pi_t) = r_t(\pi_t) \) to hold. When note issue is unrestricted, banks can always accomplish this simply by issuing notes, so no panics are possible.

We also observe that the equilibrium just derived is Pareto optimal. This follows from proposition 5.6 of Balasko and Shell (1980) and the fact that, in the equilibrium derived,

\[
\lim_{T \to \infty} \inf_{t=1}^T R_t = 1.
\]

Thus, when banks are note issuing, this economy has an equilibrium with a number of desirable properties.

5. Empirical Implications

In section IV we shall compare the predictions of the model with Canadian and U.S. data. As a prelude to doing so, we now mention several implications of the model. All of the implications we derive concern nominal interest rates and nominal quantities of bank assets, liabilities, and circulating media of exchange. Our focus on nominal quantities reflects the lack of adequate monthly price indices to be used as deflators for the dollar-denominated time series.

First, the model implies that, under an elastic currency system, there should be no seasonal variation in nominal interest rates.\(^\text{12}\) Second, the model implies that as much as 100 per cent of withdrawals can be accommodated by issuing notes. Thus the model is consistent with little or no variation in cash reserves (seasonally or otherwise). Third, the model predicts that the quantity of circulating media of exchange will be greatest when withdrawal demand is greatest (as implied by equation (4)).

To derive the remaining implications, we measure quantities in terms of their ‘end-of-period’ levels. The per capita quantity of circulating media of exchange (currency \textit{in circulation} plus banknotes) at the end of period \( t \) is \( \pi_t r^m_t(\pi_t) d_t p_t / p_{t+1} \) in real terms, since banks give currency and/or notes to movers at \( t \) with a real value of \( \pi_t r^m_t(\pi_t) d_t p_t / p_{t+1} \), and these assets earn the gross return \( p_{t+1} / p_t \) between \( t \) and \( t + 1 \). Therefore, from (3), (6), and \( R_t = p_{t+1} / p_t \), the \textit{nominal} value of circulating media at \( t \) is \( \pi_t r^m_t(\pi_t) d_t / p_{t+1} = \pi_t \beta x / (1 + \beta) p_t \). The average quantity of currency plus notes in circulation is therefore given by

\[
\bar{C}_t = [\beta x / (1 + \beta) p_t] \int_0^1 \pi f_t(\pi) d\pi; \quad t = o, e.
\]

Since \( p_o < p_e \) and \( F_o \geq F_e \) in the sense of first-order stochastic dominance, it follows that \( \bar{C}_0 > \bar{C}_e \); that is, the quantity of circulating media of exchange will be highest in seasons where credit demand and average liquidity needs are greatest.

\(^{12}\) Strictly speaking, the model implies not only constant, but zero nominal interest rates. Obviously we do not take this implication literally, since positive nominal rates would arise if there were any costs of intermediation or binding legal restrictions on banks.
With respect to total loans, nominal per capita loans at \( t \) are nominal deposits less nominal reserves; that is,

\[
L_t = \frac{\beta x}{(1 + \beta) p_t} - M; \quad t = o, e.
\]

Then \( p_e > p_o \) implies \( L_o > L_e \), so that loans also increase in seasons of high credit demand and high average liquidity need. Finally, end-of-period deposits at \( t \) are simply the nominal value of beginning-of-period deposits, \( \frac{\beta x}{(1 + \beta) p_t} \), less nominal withdrawals, \( \pi_t \frac{\beta x}{(1 + \beta) p_t} \). Therefore, the total per capita quantity of deposits at the end of period \( t \), in dollar terms, is given by \( D_t = \frac{\beta x (1 - \pi_t)}{(1 + \beta) p_t} \). Average deposit levels satisfy

\[
\bar{D}_t = \left[ \frac{\beta x}{(1 + \beta) p_t} \right] \int_0^1 (1 - \pi) f_t(\pi) d\pi
\]

\[
= \left[ \frac{\beta x}{(1 + \beta) p_t} \right] \int_0^1 F_t(\pi) d\pi; \quad t = o, e.
\]

As \( p_e > p_o \) and \( F_e(\pi) \geq F_o(\pi) \forall \pi \) hold, then \( \bar{D}_e \geq \bar{D}_o \) is possible. Average nominal deposits will tend to be largest in odd periods if seasonal variations in credit demand are large relative to seasonal variations in average liquidity needs, and conversely.

III. An Inelastic Currency Regime

We now examine the operation of this economy when banknote issues are prohibited. A prohibition on note issues is intended to capture some of the regulations in force under the National Banking System (1863–1913) in the United States. During this period all banknotes were fully backed by government bonds and hence were de facto liabilities of the government. A national bank could not issue notes against general assets, and its reserve of federal government bonds (and the illiquidity of other assets) constrained its note issue. Moreover, the mechanics of note issue under the National Banking System (see Champ 1990 for a complete description) precluded banknote issues from being used to forestall panics. While U.S. banks apparently did make efforts to expand their note issues during panics (see figure 10 or Stevens 1894, 139), the mechanism for issuing notes did not permit them to ‘get out into circulation until too late to do much good’ (Laughlin 1912, 138). This view is supported by Secretary of the Treasury Carlisle’s (1894, LXXV) statement that ‘under the present laws, which do not authorize the Treasury Department to prepare and hold a reserve of blank notes ready for immediate delivery immediately upon application, from thirty to sixty days must ordinarily elapse before the issue can be made. Thus, the inducement to take out circulation when business necessities are greatest is very small, if it exists at all, and even if applications are made the circulation will probably not be secured until too late to afford relief.’ Such a delay was too long a period of time to permit banknote issues to have much impact in
responding to panic conditions. Thus a prohibition of note issues approximately captures an essential aspect of the National Banking System.

1. Equilibrium return schedules
When note issues are prohibited, banks operate as described in section II.2, subject to the additional restriction that \( b_t(\pi_t) \equiv 0 \). Again, competition for deposits will force zero profits in equilibrium. Moreover, this same competition will force equilibrium return schedules to be chosen to maximize the (indirect) expected utility of depositors, taking their savings behaviour as given, subject to (1) and (2) at equality, \( b_t(\pi_t) \equiv 0 \), and non-negativity restrictions. Then in equilibrium, \( r^*_t(\pi_t) \), \( r_t(\pi_t) \), \( \alpha_t(\pi_t) \), and \( \gamma_t \) must be chosen to maximize

\[
\ln \left[ x/(1 + \beta) \right] + \beta \int_0^1 \pi \ln \left[ r^*_t(\pi) \beta x/(1 + \beta) \right] f_\pi(\pi) d\pi
\]

\[
+ \beta \int_0^1 (1 - \pi) \ln \left[ r_t(\pi) \beta x/(1 + \beta) \right] f_\pi(\pi) d\pi,
\]

subject to

\[
\pi r^*_t(\pi) \leq \alpha_t(\pi) \gamma_t p_{t+1}/p_t
\]

\[
(1 - \pi)r_t(\pi) \leq [1 - \alpha_t(\pi)] \gamma_t p_{t+1}/p_t + (1 - \gamma_t)R_t,
\]

\( \gamma_t \in [0, 1] \), and \( \alpha_t(\pi) \in [0, 1] \), \( \forall \, \pi \). This maximization, as before, is performed taking \( R_t \) and \( p_{t+1}/p_t \) as given. The solution to this problem satisfies (9) and (10) as equalities, along with

\[
\alpha_t(\pi_t) \leq \pi_t \left\{ 1 + [(1 - \gamma_t)/\gamma_t]R_t p_t/p_{t+1} \right\},
\]

where equality obtains if \( \alpha_t(\pi_t) < 1 \), and

\[
\gamma_t = 1 - \int_{\pi_t}^1 F_\pi(\pi) d\pi.
\]

In (12), \( \pi_t^* \) is defined to be the value of \( \pi_t \) that satisfies (11) as an equality with \( \alpha_t(\pi_t^*) = 1 \); that is,

\[
\pi_t^* \equiv \left\{ 1 + [(1 - \gamma_t)/\gamma_t]R_t p_t/p_{t+1} \right\}^{-1} \equiv g(\gamma_t, I_t),
\]

where we henceforth let \( I_t \equiv R_t p_t/p_{t+1} \) denote the (gross) nominal rate of interest.

It is apparent that for \( \pi_t \leq \pi_t^* \), \( \alpha_t(\pi_t) \leq 1 \), while if \( \pi_t \geq \pi_t^* \), \( \alpha_t(\pi_t) = 1 \). Then, when \( \pi_t \geq \pi_t^* \) holds, the banking system exhausts its cash reserves. Consistent with Noyes (1909), we associate this event with a panic, and \( r^*_t(\pi_t) = r_t(\pi_t) \forall \pi_t \leq \pi_t^* \), while \( r^*_t(\pi_t) < r_t(\pi_t) \) for \( \pi_t > \pi_t^* \). Thus, in a panic those agents needing liquidity
(movers) suffer relative to agents who do not (non-movers). This could be regarded as a disruption of the transactions process. Notice also that banks respond optimally to this ‘disruption,’ treating all movers alike. In particular there is no ‘sequential service’ constraint in effect.

Using the definition of $\pi^*_t$ in (13), we note that the bank’s optimal reserve liquidation strategy (equation (11)) can be written as

$$\alpha_t(\pi_t) = \min \left[ \frac{\pi_t}{\pi^*_t}, \ 1 \right]. \quad (11')$$

We can also succinctly characterize the optimal choice of the reserve-deposit ratio. In particular, we define the function $H_t: [0, 1] \to [0, 1]; \ t = o, e$, by

$$H_t(x) = \int_x^1 F_t(\pi) d\pi.$$  

Then (12) can be written as

$$1 - \gamma_t = H_t[g(\gamma_t, I)]; \quad t = o, e. \quad (14)$$

Equation (14) is depicted diagrammatically in figure 1. It is easy to show that $H_t[g(\gamma, I)]$ is a decreasing function of $\gamma$ and is concave in $\gamma$ if $I \geq 1$. In addition,
$H_t[g(0, I)] = H_t(0) < 1$, and $H_t[g(1, I)] = H_t(1) = 0$, $\forall I$. Finally, the slope of $H_t[g(\gamma, I)]$ exceeds one in absolute value at $\gamma = 1$ $\forall I > 1$ and is equal to one in absolute value if $I = 1$. It follows that, if $I_t = 1$, (14) is satisfied only by $\gamma_t = 1$. If $I_t > 1$ holds (the nominal interest rate is positive), then (14) has two solutions. It is easy to check that the interior solution solves the optimization problem that characterizes an equilibrium.

From (13) and the definition of $H_t(\cdot)$, it is evident that an increase in $I$ shifts $H_t[g(\gamma, I)]$ upward, as depicted in figure 1. Thus, the optimal choice of $\gamma_t$, as defined by (14), is decreasing in $I_t$. We summarize these results by saying that the optimal reserve-deposit ratio is given by $\gamma_t = \gamma_t(I_t)$; with $\gamma_t(1) = 0$ and $\gamma_t^t < 0$; $t = o, e$.

2. General equilibrium

We now describe a general equilibrium for this economy when note issues are prohibited. We note first that, in equilibrium, loans must be made in positive quantities. Then, since $\gamma_t(1) = 1$, it follows that $I_t > 1$ must hold $\forall t$; that is, in equilibrium nominal interest rates are always positive.

As in section II.4, in equilibrium net generational savings equal real balances, and all real balances are held as cash reserves by banks, so that (5) and (6) hold. However, $\gamma_T$ now satisfies (14), so (5), (6), and (14) are the equilibrium conditions of this economy. As before, we confine attention to periodic equilibria satisfying $R_t = R_t, p_t = p_t, \text{ and } \gamma_t = \gamma_t \forall t$, where $i = o$ for $t$ odd and $i = e$ for $t$ even.

In order to characterize such equilibria, we can use (5) and (6) to eliminate $R_t, p_t, p_{t+1}$ from (13), obtaining

$$\pi_o^* = \beta x \gamma_e)/(\beta x \gamma_e + y_o)$$  \hfill (15)

$$\pi_e^* = \beta x \gamma_o)/(\beta x \gamma_o + y_e).$$  \hfill (16)

Then, using (15), (16), and $\pi_t^* = g(\gamma_t, I_t)$ in (14) yields

$$1 - \gamma_o = H_o[\beta x \gamma_e + y_o]$$  \hfill (17)

$$1 - \gamma_e = H_e[\beta x \gamma_o + y_e].$$  \hfill (18)

Here, (17) and (18) determine the equilibrium levels of $\gamma_o$ and $\gamma_e$.

Figure 2 depicts the loci defined by equations (17) and (18). It is straightforward to show that the locus defined by (17) passes through the points $(1 - H_o(0, 0))$ and $(1 - H_o[\beta x/(\beta x + y_o)], 1)$, while the locus defined by (18) includes the points $(0, 1 - H_e(0))$ and $(1, 1 - H_e[\beta x/(\beta x + y_e)])$. It is also possible to show that the locus defined by (17) is everywhere steeper than the locus defined by (18). (See Champ, Smith, and Williamson 1994 for formal proofs of these assertions.) The following result is then immediate.

**Proposition 1.** (a) Equations (17) and (18) have a unique solution $(\gamma_o^*, \gamma_e^*)$. This solution satisfies $1 > \gamma_o^* > 0; \ t = e, o$. (b) $y_o \gamma_o^* > y_e \gamma_e^*$ holds.
This economy, then, has a unique periodic equilibrium. Panics occur in this equilibrium if and only if $\pi_i > \pi^*_e$, which corresponds to a situation in which banks exhaust their reserves. The equilibrium values $\pi^*_i$ are obtained by substituting $\gamma^*_o$ and $\gamma^*_o$ into (15) and (16). Part (b) of proposition 1 implies that $\pi^*_o < \pi^*_e$. Thus panics occur whenever withdrawal demand exceeds a critical level. This critical level is lowest in periods of greatest seasonal pressure on financial markets. Therefore, as argued by Andrew, Sprague, and Goodhart, smaller shocks are required to cause a panic in periods when the most seasonal pressure is placed on the banking system. The probability of a panic at $t$ is $1 - F_i(\pi^*_e)$. Then, since $\pi^*_e > \pi^*_o$ and $F_o \geq F_e$ in the sense of first-order stochastic dominance, panics are also most probable in periods of peak seasonal pressures.

Perhaps not surprisingly, it is the case that $\gamma^*_e \equiv \gamma^*_o$ can hold. If $\gamma^*_o < \gamma^*_e$, banks hold least reserves at the time heavy withdrawals are most likely. Sprague (1910) argues that this was in fact observed behaviour, although in contrast to what Sprague clearly implies, this does not indicate that banks were ignoring the probability of heavy withdrawal demand. If $\gamma^*_e < \gamma^*_o$, reserves are highest in periods when large withdrawals are most likely. The result $\gamma^*_e \leq \gamma^*_o$ will obtain if $y_o - y_e$ is sufficiently small, given $F_o(\cdot)$ and $F_e(\cdot)$, while $\gamma^*_o \leq \gamma^*_e$ will hold if $\max |F_e(\pi) - F_o(\pi)|$ is sufficiently small, given $y_o > y_e$. 
Again, having obtained $\gamma_o^*$ and $\gamma_e^*$, the equilibrium inverse price levels $p_o^*$ and $p_e^*$ are given by (6). If $\gamma_o^* \geq \gamma_e^*$, then $p_o^* \geq p_e^*$. Restrictions on note issues can then reverse the seasonal pattern of prices relative to the elastic currency regime.

It is also of interest to consider the seasonal patterns of nominal and real interest rates.

**Proposition 2.** $I_o^* > I_e^*$ and $R_o^* > R_e^*$ hold, where a ‘*’ denotes an equilibrium level under the inelastic currency regime.

(For a proof of proposition 2, see Champ, Smith, and Williamson 1994.) Proposition 2 asserts that currency inelasticity will allow seasonal pressures on financial markets to be reflected in seasonal variation in nominal interest rates. This is consistent with Lockhart’s (1921a, 160) argument that ‘our rigid reserve system and our inelastic bank note currency were . . . the principal reasons’ for the relatively large historical fluctuations in nominal interest rates.

Proposition 2 and the fact that $1 - F_o(\pi_o^*) > 1 - F_e(\pi_e^*)$ imply that high nominal interest rates reflect a high probability of a panic. Positive nominal interest rates also indicate that the equilibrium obtained here cannot be Pareto optimal. In particular, reserves are free to create, but agents perceive a positive opportunity cost to holding them. Moreover, the consumption allocations of movers and non-movers are different in general. The possibility of panics and lack of Pareto optimality indicates that the inelastic currency regime has few desirable characteristics. We should note, however, that the equilibria of the elastic and inelastic currency regimes are not Pareto comparable, and that there are ‘gainers’ under the inelastic currency regime.

3. Currency premia and panics

Currency premia were important observed features of panics, as noted by Andrew (1908a; 292–3), Noyes (1909), and Sprague (1910, 56–7, 187). A currency premium exists if currency is worth more than deposits having an equal face value. Such a premium arises in our model when panics occur, as we now demonstrate.

Imagine that either movers or non-movers can withdraw their deposits at date $t$, receiving a payment (per unit withdrawn) with a time $t + 1$ value of $r_i^m(\pi_t)$ in state $\pi_t$. This payment is obtained in currency, for which movers have use and non-movers do not. Thus non-movers will exchange any currency they obtain for claims to the deposits of movers. They will be indifferent between doing so and leaving their deposit until time $t + 1$ if and only if they obtain a real value of $r_i(\pi_t)$ per unit withdrawn. Similarly, movers will be indifferent between obtaining currency from their bank or from non-movers if and only if they can obtain currency with a real value of $r_i^m(\pi_t)$ at $t + 1$ in exchange for deposits with a real value of $r_i(\pi_t)$ at $t + 1$. Thus, after $\pi_t$ is realized, currency must exchange for deposits at the rate $q_i(\pi_t) \equiv r_i(\pi_t)/r_i^m(\pi_t)$ and a currency premium exists if $q_i(\pi_t) - 1$ is positive. We have
\[ q_t(\pi_t) = \begin{cases} 
1 & \pi_t \leq \pi^*_t \\
I_t(1 - \gamma_t)\pi_t/\gamma_t(1 - \pi_t) & \pi_t > \pi^*_t 
\end{cases} \]

Given the definition of \( \pi^*_t \) from (13), a currency premium arises if and only if banks suspend convertibility, which occurs when \( \pi_t > \pi^*_t \).\(^{13}\)

During banking panics, if agents in the model could produce currency substitutes, they would, since there are opportunities for increasing the expected utility of depositors by doing so (consumption in the second period of life is higher for non-movers when a panic occurs). In fact, if emergency currency could be created that was a perfect substitute for fiat currency, the currency would be elastic, and banking panics would not occur. In practice, the restrictions on note issue in the United States to some extent could be circumvented; the creation of currency substitutes during panics is discussed by Andrew (1908b) and Friedman and Schwartz (1963). These currency substitutes however, were not perfect substitutes for fiat currency in making all transactions. In particular, clearinghouse certificates were of limited use in executing transactions over long distances, especially as regards crop moving.

4. Remarks

Panics in this model occur purely because of the exhaustion of bank reserves. Sprague (1910a, 222) asserts that, at least in the panic of 1873, suspension occurred in New York not because of any perception of insolvency of the banks, but because of the imminent exhaustion of their reserves. Thus, our description seems to capture a real historical phenomenon.

The adverse consequences of panics in our model derive entirely from the fact that they disrupt interlocution exchange. Sprague (1910, 200) claims that in 1893 suspensions ‘deranged the exchanges between different parts of the country,’ and he also argues (71) that if trade were ‘purely local’ in nature, suspension would have had only minor consequences, since in this case transactions could have been accomplished with checks or other ‘local’ currency substitutes. (This point is reinforced by the discussions in Andrew 1908b, especially 515; and Lockhart 1921b, 228, 233–4.) Again this seems consistent with our description.

In this model, panics are in some sense simply a symptom of the larger problems caused by currency inelasticity. These problems are reflected in the presence of a positive nominal interest rate, which is indicative of a permanent inefficiency. Interestingly, this appears to have been Laughlin’s (1912, 129) view of the National Banking System: ‘it is probable that the evils experienced from this unsatisfactory bank currency in periods of panics are not nearly as great as those which result from the unsatisfactory working of the system year in and year out.’

\(^{13}\) The argument just given depends on the assumption that only lenders can contact the bank after \( \mu_t \) is realized (borrowers and lenders never meet). If borrowers could recontact the bank, they would potentially have an incentive to acquire some currency at the beginning of a period in the hope of buying back their loan at a distant if \( \pi_t > \pi^*_t \). We avoid this by assuming borrowers are not able to recontact the bank, or accomplish the same objective by making side payments to lenders. We observe that this could also be avoided by making additional assumptions on the distributions \( F_t(\cdot) \). Here, however, we adopt the simpler device of ruling out certain meetings.
5. Empirical implications

Some empirical implications of the model have already been derived. First, panics are most likely in periods of greatest seasonal pressure on financial markets. That this is observed in the data is a major theme of Sprague (1910) and Goodhart (1969). Second, nominal interest rates will vary seasonally, being highest in periods of seasonal pressures. To derive further implications we proceed as in section 1.5, and we derive expressions for end-of-period values for currency in circulation, reserves, loans, and deposits, all in nominal terms.

End-of-period bank reserves are simply beginning-of-period reserves \( (M) \) less liquidated reserves \( [\alpha_t(\pi_t)M] \). Therefore, nominal end-of-period bank reserves are given by

\[
K_t(\pi_t) = [1 - \alpha_t(\pi_t)]M = \begin{cases} 
M(1 - \pi_t/\pi_t^*) & \pi_t < \pi_t^* \\
0 & \pi_t \geq \pi_t^*
\end{cases}
\]

Since, \( \pi_e^* > \pi_o^* \), end-of-period reserves will be no higher in odd than in even periods for any value of \( \pi_t \), and they will be strictly lower for some \( \pi_t \). Thus average reserves will be lowest in periods when seasonal demands on the banking system are greatest. Furthermore, since currency in circulation at \( t = t \) is \( \alpha_t(\pi_t)M = M - K_t(\pi_t) \), the average stock of currency in circulation will be greatest in odd periods.

With respect to deposits, the end-of-period nominal value of deposits is simply the beginning-of-period nominal value \( [\beta x/(1 + \beta)p_t] \) less withdrawals \( [\alpha_t(\pi_t)M] \). Thus, if \( D_t \) is end-of-period deposits at \( t \), then

\[
D_t(\pi_t) = \begin{cases} 
\beta x/(1 + \beta)p_t - M\pi_t/\pi_t^* & \pi_t \leq \pi_t^* \\
\beta x/(1 + \beta)p_t - M & \pi_t \geq \pi_t^*
\end{cases}
\]

If \( p_o^* \geq p_e^* \), then \( D_o(\pi_t) \leq D_e(\pi_t) \), \( \forall \pi_t \), and deposits will be smallest (state by state) in odd periods. If \( p_e^* > p_o^* \), this conclusion need not hold.

It is also of some interest to consider the end-of-period reserve-deposit and currency-deposit ratios. The reserve-deposit ratio is \( K_t(\pi_t)/D_t(\pi_t) \) at \( t \), which will attain its minimum when \( \pi_t \geq \pi_t^* \) (during panics). The currency-deposit ratio if \( \pi_t < \pi_t^* \) (a panic does not occur) is given by

\[
[M - K_t(\pi_t)]/D_t(\pi_t) = \pi_t^*/(\pi_t^* - \pi_t^*)
\]

In the event of a panic \( (\pi_t \geq \pi_t^*) \), the currency-deposit ratio is

\[
[M - K_t(\pi_t)]/D_t(\pi_t) = \gamma_t^*/(1 - \gamma_t^*)
\]

Since \( \pi_t^*/(\pi_t^* - \pi_t^*) < \gamma_t^*/(1 - \gamma_t^*) \), the currency-deposit ratio will be highest during panics. As noted by Friedman and Schwartz (1963), this is true historically. Indeed, an outside observer, armed only with observations on monetary aggregates and a knowledge of when panics occurred, might be tempted to conclude that a
panic occurs when the currency-deposit ratio exceeds a critical value. Of course, in the model a causal interpretation along these lines would not be warranted.

Finally, nominal loans in the model are nominal deposits less nominal reserves, which, as before, is given by $\beta x (1 + \beta) p_t - M = L_t$. Then $L_o > L_e$ if and only if $p_o^* < p_e^* (\gamma_e > \gamma_o^*)$. This seems to correspond to what Sprague (1910) claimed was true historically. However, if $p_o^*_o \geq p_e^*$, then $L_o \leq L_e$ can hold.

IV. COMPARISON OF THE TWO REGIMES

1. Policy aspects

We have seen that an elastic currency regime supports a Pareto optimal equilibrium in which panics are avoided. An inelastic currency regime must have $I_1 > 0 \forall t$ and hence cannot support an optimal equilibrium. Moreover, it permits panics to occur.

There was some reason for the organizers of the National Banking System to believe that adequate incentives were provided within it to achieve a reasonable degree of currency elasticity. (See Champ 1990 for an argument to this effect.) By the late 1800s, however there was substantial agreement that this was not the case. West (1977, 43–5) asserts that ‘after 1893 the business and financial community was nearly unanimous in its desire to abolish bond-secured currency and issue a new national banknote secured by the assets of the issuing bank’ and describes several proposals to create an ‘asset currency’ along Canadian lines (see also Goodhart 1969, 30; Sprague 1910, 1910a, b, c). Many of the ingredients of such proposals were incorporated into the Aldrich-Vreeland Act (which permitted ‘emergency’ note issues against general assets) and ultimately in the Federal Reserve Act. It is interesting that Friedman and Schwartz (1963, 172) argue that in 1914 ‘the Aldrich-Vreeland Act provided an effective device for solving a threatened interconvertibility crisis.’ This is consistent with the implications of our model.

In our model it is useful to note that a central bank, armed with sufficient information, could create an appropriate degree of currency elasticity, even in the presence of restrictions on private note issues. That is, if the central bank could observe $\pi_t$, a policy rule that made the stock of fiat currency (through open market operations, lump-sum transfers, or ‘discount window’ policy) appropriately contingent on $\pi_t$ and the season could replicate the equilibrium achieved in the elastic currency regime.

2. Empirical aspects

We now contrast the predictions of our model with monthly banking (and interest rate) data over the period 1880–1910 (1902–14) for the United States and Canada. Implicit in our approach is the assumption that the United States and Canada were essentially identical (in per capita terms) economically, except for the differences in their banking systems, and that they experienced the same underlying disturbances. All banking data (that appear in levels) are in nominal terms, since there are no adequate monthly price indices. The Canadian data are quite comprehensive,
consisting of monthly balance sheet data for all chartered banks. Except for national bank circulation (which is comprehensive), the U.S. banking data are for New York Clearinghouse banks only. Interest rates are call loans rates in New York and Montreal. (A complete description of the data appears in the appendix.)

For each time series we performed the decomposition $x_t = x_T^t + x_S^t + x_B^t$, where $x_t$ is the natural logarithm of the raw series, and $x_T^t$, $x_S^t$, and $x_B^t$ are trend, seasonal, and cyclical components, respectively. The decomposition obtained $x_T^t$ by use of a Hodrick- Prescott filter. The remaining percentage deviations from trend were regressed on monthly dummies to obtain $x_S^t$, and $x_B^t$ constitutes the residual. We are now interested in examining the behaviour of the seasonal and cyclical components of these series.

As we discussed, seasonal changes in odd periods in the model are intended to capture recurring seasonal pressures on financial markets created by higher than average demands for credit and liquidity during the autumn crop-moving season. The model predicts that, for Canada (the elastic currency regime), currency circulation should be high during the fall. Indeed, this prediction is borne out in figure 3, which indicates considerable seasonal elasticity in the stock of banknotes in circulation, with a peak in the autumn months. Correspondingly, as is consistent with the model, figure 3 indicates almost no seasonal variation in U.S. national banknote circulation.

The model also predicts that Canada should display no seasonal variation in nominal interest rates, while there should be seasonal variation in these rates in the United States (with nominal rates being highest in the fall). Figure 4 indicates

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14 Obviously the Canadian and U.S. (New York) data are not entirely comparable. If Bagehot's (1873, 31) view is adopted that what is essential 'in time of apprehension' is the behaviour of 'whatever bank or banks keep the ultimate banking reserve of the country,' however, then clearly the New York Clearinghouse banks are of paramount interest in the United States. Owing to the small numbers of banks in Canada, the behaviour of the Canadian data is also likely to be dominated by the largest banks, who could be viewed as playing this role. Thus we think a comparison of these series is appropriate. Parenthetically, we have also examined the same series for the Chicago, New Orleans, and St Louis Clearinghouse banks. They appear qualitatively similar to those for New York.

15 The Hodrick-Prescott filter (see Hodrick and Prescott 1990; Kydland and Prescott 1990) involves a smoothing parameter, which we set equal to 14,400, based on a suggestion by Ed Prescott. We experimented with values of this parameter as low as 1,600, with no important changes in figures 3–8. We also experimented with other detrending procedures, regressing on time polynomials and log differencing, for example. The former resulted in no apparent changes in seasonal patterns; the latter preserved the seasonal patterns we report, but also induced some additional seasonal variation in the spring. This detrending procedure would not have altered any conclusions presented here. Furthermore, the model suggests treating periods in which suspensions occur differently from those periods without suspensions. In an attempt to account for this, we also performed seasonal regressions containing dummy variables for periods in which suspensions arose. We found that inclusion of such variables had little quantitative and no qualitative impact on our results.

16 We also examined the total stock of money in circulation in each country and observed a similar pattern to that of figure 3, with slightly more seasonal variation in the U.S. total money stock than in U.S. banknotes. Upon closer examination we also found significant seasonal variation in the circulation of Canadian dominion notes. However, given that dominion notes averaged only 15 per cent of total circulation during the time period, the Canadian chartered banks were the primary source of elasticity in the Canadian circulation.
that these predictions are supported by the data. Here, however, we should offer
two caveats. First, the predictions of the model literally concern time loans, while
the data constitute call loan rates. Since no other interest rate data are available for
Canada, there appears to be no superior method for confronting the predictions of
the model with the data. Second, there are some grounds to doubt the comparability
of the New York and Montreal call loan markets, since Montreal was a far less
important financial centre than New York. Indeed, there was less seasonal variation
in call loan rates in other U.S. cities than there was in New York (Goodhart 1969,
less seasonal variation than call loan rates in Chicago or Boston, confirming the
predictions of the model. Also, the fact that the Montreal call loan market was
thinner than that in New York need not have biased the Montreal market towards
less interest rate volatility.17

The model makes no explicit predictions about the behaviour of bank reserves
in Canada, although it is consistent with little or no seasonal variation in them.
For the inelastic currency regime (the United States) however, the model predicts
that reserves will be (on average) low in the fall. As indicated in figure 5, this is
consistent with observation. Notice that the seasonal volatility of U.S. reserves is
much greater than that in Canada, with reserves falling more than 6 per cent below
average in the autumn. Canadian reserves tend to rise slightly in the fall though
not significantly above trend. Figure 6 shows that similar patterns are observed in
the behaviour of the reserve-deposit ratio.

For Canada, the model predicts that nominal loans should be higher than average
in the autumn months, but that the seasonal pattern in nominal deposits depends
on the relative importance of credit and liquidity shocks. Figures 7 and 8 indicate a
relatively small amount of seasonal variation in both loans and deposits in Canada,
with seasonal deviations from trend of at most slightly more than 1 per cent. In
figure 7, there is a peak in loans in the fall (though the peak is not significantly
different from zero at the 5 per cent significance level), but loans are also high
on average in the spring and early summer. In the United States, there is more
seasonal variation in nominal loans and deposits than in Canada, with both being
low on average in the fall. In the model, this is consistent with a liquidity shock
in the fall, which is large relative to the shock to credit demand.

Figures 9–18 plot the business cycle components for all the quantity data.
Banking panics occurred during 1884, 1893, and 1907 in the United States (Sprague
1910). With this sample of three, one panic occurred in mid-year (1884), while the
1893 and 1907 panics were in the autumn, when the model predicts that the proba-
bility of a panic is highest. Panics show up most clearly in the business cycle
component of U.S. bank reserves (figure 12).

Under the assumption that the United States and Canada are subject to the
same shocks, we should observe large quantities of banknotes in circulation in

17 It is easy to produce examples where increasing the thinness of a market increases price vari-
ability. For example, if there is a set of quantity-setting firms producing a homogeneous product
at a constant marginal cost, who face a linear stochastic demand function, price variability in-
creases in a Nash equilibrium as the number of firms decreases.
FIGURE 3 Banknote circulation (seasonal coefficients, percentage deviation from trend)

FIGURE 4 Call loan rates (seasonal coefficients, percentage deviation from trend)
FIGURE 5  Cash reserves (seasonal coefficients, percentage deviation from trend)

FIGURE 6  Reserve-deposit ratio (seasonal coefficients, deviation from trend)
FIGURE 7  Loans (seasonal coefficients, percentage deviation from trend)

FIGURE 8  Deposits (seasonal coefficients, percentage deviation from trend)
Canada coincident with panics in the United States, given that banknote issues in Canada accommodate liquidity shocks. In figure 9 the business cycle component of Canadian banknotes in circulation is below trend during the 1884 panic but above trend during the panics of 1893 and 1907. Banknote circulation in the United States also rises in the vicinity of the panics of 1893 and 1907, but the peak note issues actually occur one or two months after the panics and two or more months later than peak note issues in Canada. This is consistent with the arguments of Laughlin (1912) and Carlisle (1894) that the mechanism for issuing notes in the United States prevented them from being useful in panics. Note also, in figures 9 and 10, that there is much more high-frequency variation in Canadian banknote circulation than in national banknotes, consistent with a greater sensitivity of the currency stock in Canada to financial shocks.

The business cycle components of reserves in New York City and the reserve ratio (figures 12 and 16) show a marked difference from those in Canada (figures 11 and 15). During all three panic periods there were very large declines in New York in both the quantity of reserves and the reserve ratio. Reserves are from 25–55 per cent below trend during the three panics. In Canada, however, there are no unusual deviations from trend in either reserves or the reserve ratio during U.S. panic periods. Note that deviations in the Canadian reserve ratio from trend are at most slightly greater than 2 per cent (figure 15), while the largest deviation in New York is more than 10 per cent from trend. This behaviour of reserves is consistent with the model’s predictions.

In figures 13 and 14 the business cycle components of nominal deposits in Canada and the United States are plotted. The relative behaviour here is similar to that of reserves. U.S. deposits display more volatility than deposits in Canada, and there are precipitous declines in deposits during the 1884 and 1893 panics in the United States. The absence of a significant decline in deposits in the United States during the 1907 panic is explained by the relatively early suspension of convertibility at that date. Deposit declines in Canada do not appear to be directly related to any of the U.S. panic dates, though relatively large declines occur following the 1884 and 1907 panics. These decreases in deposits are much smaller, in terms of percentage deviations from trend, than the declines that occurred in the United States during the 1884 and 1893 panics.

As for the loans, the plots of the business cycle components of loans in Canada and the United States (figures 17 and 18, respectively) show less average volatility in Canada than in the United States. In the United States loans either decline precipitously (1884 and 1893) at panic dates or are below trend (1907). Loans in Canada are actually above trend, though declining in 1893 and 1907.

V. SOME COMMENTS ON THE 'SEASONAL PLUS SHOCK' VIEW OF PANICS

The following view of panics, which can be attributed to Andrew, Sprague, and others, has found wide acceptance and has considerable intuitive appeal: 'Financial crises and seasonal variations were not considered to be separate phenomena.
FIGURE 9  Canadian banknotes in circulation (business cycle component)

FIGURE 10  U.S. national banknotes in circulation (business cycle component)
FIGURE 11  Canadian cash reserves (business cycle component)

FIGURE 12  New York City Clearinghouse Bank cash reserves (business cycle component)
FIGURE 13  Canadian total deposits (business cycle component)

FIGURE 14  New York City Clearinghouse Bank net deposits (business cycle component)
FIGURE 15  Canadian reserve-deposit ratio (business cycle component)

FIGURE 16  New York City Clearinghouse Bank reserve-deposit ratio (business cycle component)
FIGURE 17  Canadian total loans (business cycle component)

FIGURE 18  New York City Clearinghouse Bank total loans (business cycle component)
Financial crises were attributed, with a great deal of truth, not so much to cyclical factors as to the natural results of the recurring autumnal pressures upon the money market; ... it took only a little extra strain ... to turn tightness into distress' (Goodhart 1969, 3). More specifically, according to the 'seasonal plus shock' notion, there were large autumnal pressures on New York banks to ship cash to the interior for 'crop-moving' purposes. This recurring strain, augmented by even relatively minor additional shocks to withdrawal demand, could cause a shortage of liquidity and suspension of convertibility by New York banks.

Criticisms of the 'seasonal plus shock' view appear in Patterson (1913), Goodhart (1969), and Calomiris and Gorton (1990). Those authors argue that, even though New York banks shipped cash to the interior in the fall, interior banks shipped cash back in the form of direct lending in New York. Thus, the liquidity position of New York was not seriously impaired. Patterson (1913, 524–6) and Goodhart (1969, 5, 7, 89, 102–3) argue that loans made directly 'on account of correspondents increased enough to more than offset the flow of bankers' balances to the interior.' Thus, the argument is that net interregional funds flows were not unusually large in the fall. Calomiris and Gorton (1990), while not being specific about mechanisms, point out that net interregional funds flows were not abnormally large prior to panics.

Our analysis is very much a formal depiction of the 'seasonal plus shock' theory of panics. Notice however, that the symmetric nature of locations in our model implies that, in the model net interlocation funds flows are always zero. What is actually relevant in the model is the gross flow of funds between regions. Thus, the analysis indicates that the 'seasonal plus shock' view easily can be valid, even though net flows of funds from New York to the interior do not appear to be observed.

VI. CONCLUSIONS

This paper develops a model of bank liquidity provision in which there are both store of value and medium of exchange roles for currency.18 In addition, seasonal factors affecting credit and liquidity demand have been introduced. The result is a model in which the regulatory regime matters greatly for the possibility of panics and the existence of an optimal equilibrium. Moreover, under either an elastic or an inelastic currency regime, the model generates predictions about both the seasonal and the cyclical behaviour of banknote circulation, nominal interest rates, loans, deposits, bank reserves, and the reserve-deposit and currency-deposit ratios. An empirical investigation indicates that these predictions accord well with historical observation. The model also makes predictions about currency premia and the potential role for 'emergency' currency substitutes during panics that appear to be borne out by observation. Thus, the model permits a re introduction of monetary factors into a discussion of bank panics in an empirically plausible way.

The model also implies that, in the absence of any other organizational problems, a system of unrestricted banknote issue backed by general assets could be expected to function fairly well. It need not be the case, however, that a privately issued 'asset currency' is appropriate in the presence of other legal restrictions on a particular banking system. In Canada, which had unrestricted branching and a small number of banks, such a system appears to have worked well. In the United States, where there were branching restrictions and a vastly larger number of banks, it is possible that coordination problems among banks with respect to note issue/redemption, or informational problems for noteholders (of the type discussed by Williamson 1990), could be exacerbated, and that this would tend to weigh in favour of a uniform government currency. In fact, an argument to this effect was made by Sprague (1910a,b,c), who asserted that the introduction of an asset currency in the United States could have significant beneficial effects only if a number of these other issues also were addressed.

In conclusion, we mention two topics that we have not considered but that would be of interest in future research. First, our model abstracts from any interaction between Canada and United States during the period we study. For the issues we discuss this may be innocuous, and other authors have certainly found this abstraction useful (Noyes 1909; Rich 1989). However, there clearly were important financial linkages between Canada and the United States. For example, as Rich (1989) shows, Canadian banks were indirectly (though not directly – see Ross 1922) large lenders to the United States both seasonally and during panics. The interaction between an economy with an elastic currency and an economy without it would be another interesting topic for further investigation.

Second, there is an important distinction between banking panics and periods when the rate of bank failure is unusually high. As noted in Sprague (1910), panics could be disruptive to the transactions process, even in the absence of an abnormally large number of bank failures. Also, there could be failures of large banking institutions without the occurrence of panics, as in Canada (Williamson 1989). Depositor loss rates were quite low in both the United States and Canada in the period we examine (see Sprague 1910; Gorton 1988; Williamson 1989), so that ignoring bank failures may be a reasonable first approximation. However, there were somewhat lower loss rates in Canada than in the United States. This difference has been attributed to differences in branching regulations (see, e.g., Williamson 1989; or for a later period White 1984); but it would also be of interest to consider the possibility of currency elasticity as a contributing factor.

Appendix

Description of the Data
The source for the data analysed in this paper are detailed on the following page. All data are end-of-month observations, except call loan rates, which are monthly averages. Complete citations follow in the reference section. The transformations of the raw data are described in the paper.
<table>
<thead>
<tr>
<th>Canadian series</th>
<th>Source</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Banknote circulation</td>
<td>Breckenridge (1910, appendix VI, 290)</td>
<td>Banknote circulation of Canadian chartered banks</td>
</tr>
<tr>
<td>Call loan rate</td>
<td>Goodhart (1969, table 14, 206–20)</td>
<td>Montreal call loan rate</td>
</tr>
<tr>
<td>Cash reserves</td>
<td>Curtis (1913, 36, 38)</td>
<td>Sum of gold, subsidiary coin, and dominion notes held by chartered banks</td>
</tr>
<tr>
<td>Reserve ratio</td>
<td>Calculated</td>
<td>Cash reserves/deposits</td>
</tr>
<tr>
<td>Loans</td>
<td>Curtis (1931, 50)</td>
<td>Total loans in Canada</td>
</tr>
<tr>
<td>Deposits</td>
<td>Curtis (1931, 24–6)</td>
<td>Sum of total deposits by the public, provincial governments deposits, and dominion governments deposits</td>
</tr>
<tr>
<td>Call loan rate</td>
<td>Goodhart (1969, table 14, 206–20)</td>
<td>New York City call loan rate</td>
</tr>
<tr>
<td>Cash reserves</td>
<td>National Monetary Commission (1910, table 28)</td>
<td>Specie and legal tender notes held by New York City Clearinghouse banks</td>
</tr>
<tr>
<td>Reserve ratio</td>
<td>Calculated</td>
<td>Cash reserves/deposits</td>
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<tr>
<td>Loans</td>
<td>National Monetary Commission (1910, table 28)</td>
<td>Loans of New York City Clearinghouse banks</td>
</tr>
<tr>
<td>Deposits</td>
<td>National Monetary Commission (1910, table 28)</td>
<td>Net deposits of New York City Clearinghouse banks</td>
</tr>
</tbody>
</table>
All New York City Clearinghouse bank series are end-of-month observations taken from the original weekly data. The timing convention for the determination of months followed that of Kemmerer (1910, 13).

The shaded regions in the plots of the seasonal components of the data represent 95 per cent confidence intervals for the seasonal coefficients.

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