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Stephen D. Williamson


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SECTORAL SHIFTS, LABOR MARKET SORTING, AND AGGREGATE FLUCTUATIONS

BY STEPHEN D. WILLIAMSON

A model of sectoral reallocation is constructed where intersectoral friction is not caused by search or mobility costs. Instead, a sectoral disturbance has a negative effect on production possibilities because it reduces the value of previous sorting in the labor market. In equilibrium, a measure of sectoral dispersion is positively correlated with the unemployment rate. An increase in the incidence of sectoral disturbances increases unemployment at cyclical peaks and at cyclical troughs.

1. INTRODUCTION

Lilien observed (Lilien 1982) that, in postwar U.S. time series data, the unemployment rate is positively correlated with a measure of the dispersion in employment growth rates across sectors of the economy. Lilien's "sectoral shifts hypothesis" explained this regularity as resulting from frictions associated with the intersectoral mobility of labor. Murphy and Topel (1987), examining CPS data, argued that mobility among sectors was procyclical, and that most of the variability in unemployment seems to be accounted for by workers who stay in the same industry rather than by those who leave. This does not appear to be consistent with the sectoral shifts hypothesis. However, recent work by Loungani, Rogerson, and Sonn (1988) finds an important role for intersectoral mobility in unemployment rate fluctuations, based on a study of PSID data.

If mobility across sectors is important for aggregate fluctuations, this sectoral mobility could be caused by two types of impulses: aggregate shocks or purely sectoral shocks which sum to zero (in some sense) across sectors. Aggregate shock models with sectoral implications are constructed in Hamilton (1988), Rogerson (1986), and Weiss (1985). In these models workers face costs, in terms of time or other resources, to moving between sectors. It seems clear that the sectoral shifts hypothesis in Lilien (1982) is a theory in which purely sectoral shocks drive the business cycle. In discussions of sectoral shifts theories based on these purely sectoral shocks, there is often an appeal to search models, such as Lucas and Prescott (1974) (see Lilien 1982) or to matching models like those in Jovanovic (1984) or MacDonald (1988). However, to my knowledge there exist no general equilibrium search or matching models capable of explaining the correlation observed by Lilien.

In any model of sectoral reallocation there needs to be some friction to the

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sectoral reallocation of workers. The purpose of this paper is to construct a model with purely sectoral impulses in which the intersectoral friction does not arise from mobility costs. The model captures the following type of sectoral shifts mechanism. The labor force consists, at any point in time, of individuals with different attributes. For our purposes, the most important of these attributes are age and productivity (or skill level) in performing particular tasks. Though a person’s age and job history are essentially publicly observable, an individual generally has more information than do others about her productivity in a particular production process. Though a worker’s human capital can be inferred from her education and work experience, personal attributes may be as important, or more important, than human capital in determining productivity. For example, suppose there are two engineers with the same education, but engineer A works well in groups and poorly on her own, while engineer B works well on her own and poorly in groups. Now suppose that there two goods, x and y, and that the technology for producing x requires that engineers work in groups, while the technology for producing y requires that engineers work alone. Thus, A is relatively more productive in sector x and B is relatively more productive in sector y.

Over time, the labor force is sorted according to productivities, with individual workers finding employment in the sectors of the economy where they are relatively most productive. However, workers can only be sorted to the extent that the existing pattern of sectoral production allows it. In the example above, if y is not produced then there is no way of discovering that B is relatively more productive in producing y. Therefore, if a disturbance causes a change in the sectoral composition of production, then this reduces (in some sense) the value of information previously acquired on individual workers through sorting. The effect of a sectoral disturbance on an individual worker will increase with the age of the worker because of this effect, since older workers have generally spent longer in the sorting process. As a sectoral disturbance is associated with more misallocation of resources, aggregate output will tend to be lower than average and the unemployment rate higher than average.

The model constructed is one of two-period-lived overlapping generations, where agents have private information concerning their productivities in each sector of the economy. An overlapping generations framework with two-period-lived agents is a particularly useful apparatus for modeling the kind of process we are interested in, since it captures the necessary heterogeneity of agents across types and ages in a tractable manner in a dynamic model.

Unemployment is modeled here as it is in Rogerson (1988), Hansen (1985), and Greenwood and Huffman (1987). That is, labor is indivisible, and workers engage in lotteries; the outcome of the lottery determines whether an individual works or not. In general, workers of different observable types enter different employment lotteries with different probabilities of employment. Thus, unemployment rates differ across types of workers.

In a period when a sectoral disturbance occurs, goods consumed in positive amounts in the previous period are not consumed in the current period, and vice versa. This type of disturbance can be interpreted as a shock to preferences, technology, or endowments. In equilibrium, agents are perfectly sorted, when
young, according to their productivities in industries which produce in the current period. If a sectoral disturbance does not occur in the following period, then these same workers continue to be perfectly sorted. However, if a sectoral disturbance does occur, then old workers are not perfectly sorted; essentially, “productive” workers are indistinguishable from “unproductive” workers. Thus, when a sectoral disturbance occurs, old productive workers effectively subsidize old unproductive workers, and they therefore choose an employment lottery with a lower probability of employment. The unemployment rate of older workers is therefore higher than average when a sectoral disturbance occurs. A sectoral disturbance also causes the unemployment rate of young workers to rise, through an indirect effect on asset prices. That is, there is a capital asset which is exchanged between generations, and the price of this asset moves procyclically in equilibrium. When asset prices are low, young agents give up less in current consumption to acquire the existing stock of assets, and they therefore work with a lower probability and consume more leisure in the aggregate. Therefore, the model predicts that the unemployment rates of both young and old workers are positively correlated with a measure of sectoral dispersion.

In addition to predicting cyclical labor market behavior, the model generates secular predictions. In particular, an increase in the unconditional probability of a sectoral disturbance increases unemployment rates at cyclical peaks and at cyclical troughs. This provides an explanation for the behavior of the U.S. unemployment rate in the 1970s, when sectoral disturbances were perceived to have been relatively frequent. Also, an increase in the degree of serial correlation of sectoral disturbances increases the amplitude of fluctuations, raising the unemployment rate at cyclical peaks and lowering it at cyclical troughs.

The remainder of the paper is organized as follows. In Section 2 the model is presented, and an equilibrium is characterized in Section 3. Section 4 contains an analysis of equilibrium fluctuations, and a discussion of the model’s predictions. Some remarks on features of the model are in Section 5, and Section 6 gives a summary and conclusion.

2. THE MODEL

The population consists of overlapping generations of consumers. At the beginning of each period, \( t \), a continuum of two-period-lived agents is born, where \( t = 1, 2, 3, \ldots \). The measure of generation \( t \) is unity. There are two perishable consumption goods, 1 and 2. Agents consume good 1 when young and consume both goods when old. Each agent is endowed with \( h \) units of time when young and when old, and allocates this time endowment between work and leisure. Labor is indivisible, as in Rogerson (1988), Hansen (1985), and Greenwood and Huffman (1987a, 1987b). That is, agents can supply only one unit or zero units of labor time in either period of life, where \( h > 1 \). A member of generation \( t \) maximizes the expected value of utility,

\[
U(c'_t, c'_{t+1}, d'_{t+1}, l'_{t+1}, l''_{t+1}) = u(c'_t) + w(l'_t) + v(c'_{t+1}, d'_{t+1}) + z(l''_{t+1}),
\]
where \( c^s_t \) denotes consumption of good 1 by a generation \( s \) agent in period \( t \). Similarly, \( d \) denotes consumption of good 2 and \( l \) denotes leisure time. The functions \( u(\cdot) \) and \( v(\cdot , \cdot) \) are increasing in all arguments, twice continuously differentiable, and strictly concave. We assume that \( w(h) > w(h - 1), z(h) > z(h - 1), u'(0) = \infty, D_{11}u(0, d) = \infty, D_{21}u(c, 0) = \infty, \) and \( D_{12}v(c, d) > 0 \), for \( c, d > 0 \). Thus, agents prefer staying at home to working, they will consume positive quantities of all goods in equilibrium, and goods 1 and 2 are complements. We also assume that 
\[-[dD_{22}v(c, d)/D_{21}v(c, d)] \leq 1, \text{ for } c, d > 0.\]
This last assumption rules out perverse behavior; essentially, it assures that an increase in the wage rate of old workers does not lead to a fall in labor supplied by old workers in the aggregate (ceteris paribus).

Technologies for producing goods 1 and 2 using labor as an input are owned by perfectly competitive firms. For any of these firms to produce in period \( t \), it must employ a measure of at least \( \varepsilon \) workers, where \( 0 < \varepsilon \ll 1 \). Firms can produce goods 1 and 2 in either of two colors, green or blue. However, in any period all agents consume the same color of goods 1 and 2.

Agents differ in their ability to produce green and blue goods 1 and 2. Young agents can produce only good 1 while old agents produce only good 2. An agent can be one of four types, indexed by \( i = 1, 2, 3, 4 \), where type does not change over the agent’s lifetime. A type 1 agent can produce goods in either color, type 2 agents produce only green goods, type 3 agents produce only blue goods, and type 4 agents produce neither color. If productive, a young (old) agent produces \( y_1(y_2) \) units of good 1(2) for each unit of labor time supplied.

In each generation, the measure of type 1 agents is \( \pi \), the measure of type 2 agents is \( \pi^{1/2} - \pi \), the measure of type 3 agents is \( \pi^{1/2} - \pi \), and the measure of type 4 agents is \( 1 + \pi - 2\pi^{1/2} \), where \( 0 < \pi < 1 \). Type is private information, and firms can observe the total output of the group of workers hired, but cannot observe the output of individual workers.

Let \( \delta_t \) denote the state of the world, where \( \delta_t = g \) if agents consume green goods, and \( \delta_t = b \) if agents consume blue goods in period \( t \). Let \( \eta_t \) denote changes in \( \delta_t \), where \( \eta_t = 0 \) if \( \delta_t \neq \delta_{t-1} \), and \( \eta_t = 1 \) if \( \delta_t = \delta_{t-1} \). Given the environment and the equilibria we examine, \( \eta_t \) is the relevant state variable, and it follows a two-state Markov process, with \( P[\eta_{t+1} = 1|\eta_t] = \theta(1 - \phi) + \phi \eta_t \). Here, \( 0 < \theta < 1 \), and \( 0 \leq \phi \leq 1 \). The parameter \( \theta \) is the unconditional probability of observing \( \eta_t = 1 \), and \( \phi \) is the serial correlation coefficient.

When \( \eta = 0 \) there is a sectoral disturbance, i.e., a shift from production of one color of goods 1 and 2 to production of the other color. This disturbance can have a number of interpretations as combinations of shocks to preferences, technology, and endowments. For example, a sectoral disturbance could be simply a shift in color preference. Alternatively, the disturbance could be a change in relative factor supplies. Suppose, for example, that production of a given color good requires some other factor of production that is perishable and received each period by firms as an endowment. This endowment is either green or blue, and can be used only to produce the same color of good. A sectoral disturbance occurs when the color of the factor endowment changes from what it was last period.

Each member of generation 0 alive at \( t = 1 \) is endowed with \( K \) units of
perfectly-divisible, infinite-lived, nonaugmentable capital. A unit of capital yields a return of $r$ units of good 1 per period. Generation 0 agents consume only good 1 at $t = 1$, and they sell their entire endowment of capital, after consuming its return, in order to maximize consumption.

The basic mechanism by which sectoral shocks cause fluctuations in aggregate output, employment, and unemployment rates, would remain intact if there were only one consumption good and no capital good. Having a capital good and two consumption goods, with a complementarity in consumption for old agents performs two functions. First, this will cause the unemployment rates of young and old workers to move together in equilibrium, and second, it introduces some dynamics into the model.

3. **Equilibrium**

Rather than follow the approach of Townsend (1987, 1988), where we would first solve a planning problem, finding Pareto optimal allocations satisfying resource constraints and incentive constraints, we impose a structure of markets and contractual arrangements. This is done so that an equilibrium can be simply characterized.\(^2\)

Five types of trades occur in the model. First, intergenerational trade can be accomplished through exchanges of capital for good 1. Young agents trade good 1 for capital, consume its dividend when old, and then sell the capital to the young of the next generation. Second, old agents can exchange good 1 for good 2. Third, in period $t$, members of generation $t$ trade securities which are claims to next period’s good 1, contingent on $\eta_{t+1}$. Fourth, firms exchange goods with workers for labor. Fifth, workers engage in employment lotteries. As in Rogerson (1988), an employment lottery is preferred by a worker, ex ante, to the discrete choice of working or not working.

Notation is defined as follows:

- $c_{i}^{j}(t)$ = consumption of good 1 at time $t$ by a type $i$ young agent,
- $c_{i}^{j}(t, j)$ = consumption of good 1 at time $t$ by a type $i$ old agent, when $\eta_{t} = j$,
- $d^{i}(t, j)$ = consumption of good 2 at time $t$ by a type $i$ old agent when $\eta_{t} = j$,
- $\rho_{i}^{j}(t)$ = probability of employment for a type $i$ young agent at time $t$,
- $\rho_{i}^{j}(t, j)$ = probability of employment for a type $i$ old agent at time $t$ when $\eta_{t} = j$,
- $f_{i}^{j}(t)$ = number of securities purchased by a young type $i$ agent at time $t$. Each security is a claim to 1 unit of good 1 if $\eta_{t+1} = j$.
- $k_{i}^{j}(t)$ = capital acquired by a young type $i$ agent at time $t$,
- $\gamma_{i}^{j}(t)$ = wage faced by a type $i$ young agent at time $t$, in units of good 1,
- $\gamma_{i}^{j}(t, j)$ = wage faced by a type $i$ old agent at time $t$, in units of good 2, when $\eta_{t} = j$,
- $p_{i}^{j}(t)$ = price of a security which pays 1 unit of good 1 if $\eta_{t+1} = j$.

\(^2\) The equilibrium allocation is not Pareto optimal, in general, as the contractual arrangements in the model do not permit elements of cooperation, such as cross subsidization in labor contracts. This suboptimality should not affect the results, however. Private information would impose allocational costs associated with sectoral shocks even if the allocation were Pareto optimal.
\( q(t, j) \) = price of good 2 in terms of good 1 when \( \eta_t = j \),
\( s(t, j) \) = price of capital in terms of good 1 when \( \eta_t = j \).

In period \( t \), a type \( i \) member of generation \( t \) chooses \( c^i_j(t), c^i_{j+1}(t), d^i(t+1, j), \rho^i_j(t+1, j), f^i_j(t), k^i(t), j = 0, 1 \) to solve:

\[
\begin{align*}
(1) \quad & \max \{ u(c^i_j(t)) + w_1 - \rho^i_j(t)(w_1 - w_0) \\
& + \left[ 1 - \theta(1 - \phi) - \phi \eta_t \right] \nu(c^i_j(t+1, 0), d^i(t+1, 0)) + z_1 - \rho^i_j(t+1, 0)(z_1 - z_0) \\
& + \left[ \theta(1 - \phi) + \phi \eta_t \right] \nu(c^i_j(t+1, 1), d^i(t+1, 1)) + z_1 - \rho^i_j(t+1, 1)(z_1 - z_0) \}
\end{align*}
\]

subject to

\[
(2) \quad c^i_j(t) + p_1(t)f^i_j(t) + p_0(t)f^i_j(t) + s(t)k^i(t) \leq \rho^i_j(t)\gamma^i_j(t),
\]

\[
(3) \quad c^i_j(t+1, j) + q(t+1, j)d^i(t+1, j) \leq \rho^i_j(t+1, j)\gamma^i_j(t+1, j)q(t+1, j)
\]

\[+ [s(t+1, j) + r]k^i(t) + f^i_j(t), \]

for \( i = 1, 2, 3, 4, j = 0, 1 \). We have \( w_1 = w(h), w_0 = w(h-1), z_1 = z(h) \), and \( z_0 = z(h-1) \).

Note in (1) through (3) that, as in Rogerson (1988), the fact that preferences are separable in consumption and leisure implies that agents insure themselves perfectly against unemployment. That is, agents who enter a given employment lottery receive the same compensation, ex post, whether they work or not. Thus, if \( \gamma \) is the wage received by workers who enter the lottery and work, and \( \rho \) is the fraction of agents entering the lottery who work, then each worker receives \( \rho \gamma \), after insurance payments are made from workers to the unemployed.

Equilibrium in the capital market gives

\[
(4) \quad \sum_i k^i(t) = K, \quad t = 1, 2, 3, \ldots .
\]

and the equilibrium conditions for contingent claims markets are

\[
(5) \quad \sum_i f^i_j(t) = 0, \quad j = 0, 1, \quad t = 1, 2, 3, \ldots .
\]

Firms make wage offers to workers, allowing workers to choose the probability of employment. Workers’ ages are publicly observable, as are all their actions. For example, a particular worker’s purchases and sales of contingent claims and capital are publicly observable. In equilibrium, the following are satisfied:

(i) [zero profits] Each wage offer earns zero profits, given the workers who accept it.

(ii) [no unexploited profit opportunities] There exist no wage offers which, if offered, would earn positive profits given equilibrium wage offers and equilibrium prices.
(iii) [incentive compatibility] Given equilibrium prices, workers weakly prefer the contracts offered to their type to the contracts offered to other agent types.

Note that it is not restrictive to have firms make wage offers rather than offer contracts which specify a wage and probability of employment. In this model, in contrast to the insurance environment of Rothschild and Stiglitz (1976), for example, all agents have the same preferences and also face the same prices in other markets. Thus, firms can not induce self-selection by offering a richer contract.

3.1. Conjectured Equilibrium. The approach we will take is to make a conjecture about equilibrium behavior, imposing the zero-profit condition, (i), and to then show that the equilibrium satisfies (ii) and (iii). The conjectured equilibrium is as follows. If \( \delta_t = g \) then, among generation \( t \) agents, type 1 and type 2 agents are indistinguishable in periods \( t \) and \( t + 1 \). Also, generation-\( t \) type 3 and type 4 agents are indistinguishable in periods \( t \) and \( t + 1 \). If \( \delta_t = b \), then generation-\( t \) type 1 and type 3 agents are indistinguishable, as are generation-\( t \) type 2 and type 4 agents. Productive agents are distinguishable from unproductive agents when young, so that no productive agents work when young. If \( \eta_t = 1 \), then productive old agents are distinguishable from unproductive old agents, and no unproductive agents work. However, if \( \eta_t = 0 \), then a fraction \( \mu = \pi^{1/2} \) of each distinguishable group of old workers is not productive. The zero-profit equilibrium wage offers are then:

\[
\begin{align*}
\gamma^1_1(t) &= \gamma^2_1(t) = y_1, & \text{if } \delta_t = g, \\
\gamma^1_2(t) &= \gamma^3_1(t) = y_1, & \text{if } \delta_t = b, \\
\gamma^1_3(t + 1, 1) &= \gamma^2_3(t + 1, 1) = y_2, & \text{if } \delta_t = g, \\
\gamma^1_3(t + 1, 1) &= \gamma^3_3(t + 1, 1) = y_2, & \text{if } \delta_t = b, \\
\gamma^1_4(t + 1, 0) &= \gamma^2_4(t + 1, 0) = \gamma^3_4(t + 1, 0) = \gamma^4_4(t + 1, 0) = \mu y_2, & \text{if } \delta_t = g \text{ or } \delta_t = b.
\end{align*}
\]

Otherwise, wage offers are zero. Assuming an interior solution for the probability of employment if a wage offer is positive, first-order conditions from the optimization problem, (1) through (3), are:

\[
\begin{align*}
(6) \quad y_i u'(c_i^j(t)) &= w_i - w_0, \quad i = 1, 2 \quad \text{if } \delta_t = g, \quad i = 1, 3 \quad \text{if } \delta_t = b, \\
(7) \quad \mu y_2 D_2 v(c_3^0(t + 1, 0), d^i(t + 1, 0)) &= z_1 - z_0, \quad i = 1, 2, 3, 4, \\
(8) \quad -p_1(t) u'(c_3^1(t)) + [\theta(1 - \phi) + \phi \eta_t] D_1 v(c_3^1(t + 1, 1), d^i(t + 1, 1)) &= 0, \quad i = 1, 2, 3, 4, \\
(9) \quad -p_0(t) u'(c_3^1(t)) + [1 - \theta(1 - \phi) - \phi \eta_t] D_1 v(c_3^1(t + 1, 0), d^i(t + 1, 0)) &= 0, \quad i = 1, 2, 3, 4,
\end{align*}
\]
\[\begin{align*}
(10) \quad -s(t)u'(c_y(t)) + & [\theta(1 - \phi) + \phi \eta_t]s(t + 1, 1) + r] D_1 v(c_p^i(t + 1, 1), d(t + 1, 1)) \\
& + [1 - \theta(1 - \phi) - \phi \eta_t]s(t + 1, 0) + r] D_1 v(c_p^i(t + 1, 0), d(t + 1, 0)) = 0, \\
& i = 1, 2, 3, 4, \\
(11) \quad q(t+1, j) D_1 v(c_p^i(t+1, j), d(t+1, j)) + D_2 v(c_p^i(t+1, j), d(t+1, j)) = 0, \\
& i = 1, 2, 3, 4, \quad j = 0, 1.
\end{align*}\]

Note that, if a wage offer is zero for a particular type of agent, then that agent type chooses an employment probability of zero.

Equations (6) through (9) and (11) imply that the equilibrium contingent claims prices are:

\[\begin{align*}
(12) \quad p_0(t) &= [1 - \theta(1 - \phi) - \phi \eta_t]y_1(z_1 - z_0)/\mu y_2(w_1 - w_o) q(t + 1, 0), \\
(13) \quad p_1(t) &= [\theta(1 - \phi) + \phi \eta_t]y_1(z_1 - z_0)/y_2(w_1 - w_o) q(t + 1, 1).
\end{align*}\]

Given that \(u(\cdot)\) and \(v(\cdot, \cdot)\) are strictly concave, (6) through (9) and (11) imply that there is perfect pooling among agents in each generation. That is,

\[\begin{align*}
(14) \quad c_p^i(t) &= c_y(t), \\
(15) \quad c_p^i(t + 1, j) &= c_p(t + 1, j), \\
(16) \quad d(t + 1, j) &= d(t + 1, j),
\end{align*}\]

for \(i = 1, 2, 3, 4, j = 0, 1, \) and \(t = 1, 2, 3, \ldots\). Perfect pooling then implies, from (2), (3), and (14) through (16), that

\[\begin{align*}
(17) \quad p_0^i(t)(w_1 - w_o) + [\theta(1 - \phi) + \phi \eta_t](z_1 - z_0)p_0^i(t + 1, 1) + [1 - \theta(1 - \phi) \eta_t] \\
\times (z_1 - z_0)p_0^i(t + 1, 0) = [1 - \theta(1 - \phi) - \phi \eta_t](z_1 - z_0)p_0^i(t + 1, 0)
\end{align*}\]

for \(i = 1, 2, \) and \(j = 3, 4\) if \(\delta_t = g\), and for \(i = 1, 3, \) and \(j = 2, 4,\) if \(\delta_t = b\). Therefore, given (14) through (17), expected utility is the same at the time of birth for all members of a given generation. The conjectured equilibrium is therefore incentive compatible, i.e. (iii) holds. Also, firms can offer no alternative contracts that would earn positive profits. Only in the case where \(\eta_t = 0\) are agents not perfectly sorted according to productivity, and it is only old agents who are not sorted. Firms could potentially make positive profits when \(\eta_t = 0\) only by offering a wage to older workers higher than the equilibrium wage and attracting a fraction of productive workers greater than \(\mu\). But, it is impossible for firms to obtain this more favorable mix of workers, since any contract preferred by productive agents would be preferred by unproductive agents to existing wage offers.

3.2. Stationary Equilibrium. Attention will be restricted to the equilibrium where prices depend only on \(\eta_t\). Given (6) through (17), equilibrium prices and quantities are the solution to (18) through (25).
\begin{align*}
\text{(18)} & \quad y_1 u'(\mu \rho_y(1)y_1 - s(1)K) = w_1 - w_0, \\
\text{(19)} & \quad y_2 D_2 v(\{s(1) + r\}K, \mu \rho_o(1)y_2) = z_1 - z_0, \\
\text{(20)} & \quad y_1 u'(\mu \rho_y(0)y_1 - s(0)K) = w_1 - w_0, \\
\text{(21)} & \quad \mu y_2 D_2 v(\{s(0) + r\}K, \mu \rho_o(0)y_2) = z_1 - z_0, \\
\text{(22)} & \quad -q(0)D_1 v(\{s(0) + r\}K, \mu \rho_o(0)y_2) + D_2 v(\{s(1) + r\}K, \mu \rho_o(1)y_2) = 0, \\
\text{(23)} & \quad -q(0)D_1 v(\{s(0) + r\}K, \mu \rho_o(0)y_2) + D_2 v(\{s(0) + r\}K, \mu \rho_o(0)y_2) = 0, \\
\text{(24)} & \quad -s(1)u'(\mu \rho_y(1)y_1 - s(1)K) + \theta(1 - \phi)D_1 v(\{s(1) + r\}K, \mu \rho_o(1)y_2) \\
& \quad + (1 - \theta)(1 - \phi)D_1 v(\{s(0) + r\}K, \mu \rho_o(0)y_2) = 0, \\
\text{(25)} & \quad -s(0)u'(\mu \rho_y(0)y_1 - s(0)K) + \theta(1 - \phi)D_1 v(\{s(1) + r\}K, \mu \rho_o(1)y_2) \\
& \quad + [1 - \theta(1 - \phi)]D_1 v(\{s(0) + r\}K, \mu \rho_o(0)y_2) = 0.
\end{align*}

Equations (18) through (25) solve for \(\rho_y(j), s(j), \rho_o(j), q(j), j = 0, 1\). Here, \(\rho_y(j)\) is the average employment rate of young workers and \(\rho_o(j)\) is the employment rate for old workers who work with positive probability when \(\eta_t = j\). We include all agents as members of the labor force, i.e. workers who are identifiably unproductive enter a lottery where the probability of employment is zero. Letting \(e_y(j)\) denote the average employment rate of young workers and \(e_o(j)\) the average employment rate of old workers when \(\eta_t = j\), we have \(e_y(j) = \mu \rho_y(j)\) for \(j = 0, 1\), \(e_o(1) = \mu \rho_o(1)\), and \(e_o(0) = \rho_o(0)\).

4. FLUCTUATIONS

The purpose of this section is to uncover the operating characteristics of the model. First, to show how private information is important to the results, we examine an equilibrium for a version of the model where all information is public. Next, we characterize fluctuations in the private information version of the model, and show how changes in underlying parameters affect the nature of these fluctuations.

4.1. A Model Without Private Information. Consider an environment identical to the one constructed in Section 2, but where type is publicly observed. Equilibrium wages will then be:

\[ \gamma^i(t) = y_1, \quad \text{for } i = 1, 2 \text{ if } \delta_t = g, \quad \text{and } i = 1, 3 \text{ if } \delta_t = b, \]

\[ \gamma^i(t) = 0, \quad \text{for } i = 3, 4 \text{ if } \delta_t = g, \quad \text{and } i = 2, 4 \text{ if } \delta_t = b, \]

\[ \gamma^i(t + 1, 1) = y_2, \quad \text{for } i = 1, 2 \text{ if } \delta_t = g, \quad \text{and } i = 1, 3 \text{ if } \delta_t = b, \]

\[ \gamma^i(t + 1, 1) = 0, \quad \text{for } i = 3, 4 \text{ if } \delta_t = g, \quad \text{and } i = 2, 4 \text{ if } \delta_t = b, \]
\[\gamma^i_{\delta}(t + 1, 0) = y_2, \quad \text{for } i = 1, 3 \text{ if } \delta_i = g, \quad \text{and } i = 1, 2 \text{ if } \delta_i = b,\]
\[\gamma^i_{\delta}(t + 1, 0) = 0, \quad \text{for } i = 2, 4 \text{ if } \delta_i = g, \quad \text{and } i = 3, 4 \text{ if } \delta_i = b.\]
Therefore, type 4 agents consume zero in all periods of life and, as in the private information setup, all other agent types in any generation achieve a perfect pooling solution. This assumes that all agents who receive a positive wage offer choose to work with positive probability. There then exists a stationary equilibrium which is the solution to (26) through (29).

\begin{align*}
(26) & \quad y_1 u'(\rho_y y_1/(2 - \mu) - sK) = w_1 - w_0, \\
(27) & \quad y_2 D_2 v([s + r]K, \rho_o y_2/(2 - \mu)) = z_1 - z_0, \\
(28) & \quad -qD_1 v([s + r]K, \rho_o y_2/(2 - \mu)) + D_2 v([s + r]K, \rho_o y_2/(2 - \mu)) = 0, \\
(29) & \quad -su'(\rho_y y_1/(2 - \mu) - sK) + (s + r)D_1 v([s + r]K, \rho_o y_2/(2 - \mu)) = 0.
\end{align*}

Equations (26) through (29) solve for \(\rho_y\) and \(\rho_o\), the employment rates of young and old agents (including only the agents with positive probabilities of working); \(s\), the price of capital; and \(q\), the relative price of good 2 and good 1. In contrast to the equilibrium with private information, there are no fluctuations. That is, sectoral disturbances cause fluctuations in this model because of the existence of private information and the problem this creates in sorting workers. If the pattern of sectoral production is the same in period \(t\) as it was in period \(t - 1\), then older workers are correctly sorted according to productivity. However, if sectoral production changes, then the sorting of workers which occurred in period \(t - 1\) is of no use in allocating workers to period \(t\) activities.

The equilibrium without private information is not an equilibrium with private information. Without private information, type 4 agents consume zero in all periods of life in equilibrium, and are therefore worse off than all other agent types. The equilibrium is therefore not incentive compatible with private information, i.e. it does not satisfy (iii).

4.2. Private Information. Note that the solutions to (18) through (25) and (26) through (29) are identical when \(\mu = 1\), i.e. the sorting problem disappears as the fraction of agents who are productive in all states goes to unity. To characterize the fluctuations that take place in the private information case, we look at the behavior of the system of equations (18) through (25) for \(\mu\) close to 1. That is, we study the behavior of the model when perturbed by a small amount of private information. Totally differentiating (18) through (25) and solving, we obtain the following, where all derivatives are evaluated at \(\mu = 1\):

\begin{align*}
(30) & \quad \delta s(1)/\delta \mu = -(r + s)u_{12}(1 - \theta)(1 - \phi)(w_1 - w_0)(z_1 - z_0)/v_{22} y_1 y_2 > 0, \\
(31) & \quad \delta s(0)/\delta \mu = -(z_1 - z_0)u_{12}[(w_1 - w_0)((1 - \theta)(1 - \phi)s + [1 - \theta(1 - \phi)r])]/y_1 - \phi(r + s)^2 K]/v_{22} y_2 > 0,
\end{align*}
\[ \frac{\partial \rho_y(1)}{\partial \mu} = -\rho_y + K[\partial s(1)/\partial \mu]/y_1, \]
\[ \frac{\partial \rho_y(0)}{\partial \mu} = -\rho_y + K[\partial s(0)/\partial \mu]/y_1, \]
\[ \frac{\partial \rho_o(1)}{\partial \mu} = -\rho_o - v_{12}K[\partial s(1)/\partial \mu]/v_{22}y_2, \]
\[ \frac{\partial \rho_o(0)}{\partial \mu} = -\rho_o - v_{12}K[\partial s(0)/\partial \mu]/v_{22}y_2 - v_2/v_{22}y_2, \]

where
\[ \nabla = [(w_1 - w_0)r/y_1(r + s) - (r + s)VK][\nabla_1 - w_0][r + (1 - \phi)s]/y_1(r + s) \]
\[ - \phi(r + s)VK > 0, \]

and \( V = v_{22} - (v_{12})^2/v_{11} < 0. \)

We are concerned primarily with how key variables move between states. From (30) through (35), we get
\[ \frac{\partial s(1)}{\partial \mu} - \frac{\partial s(0)}{\partial \mu} = (r + s)v_{12}\phi(z_1 - z_0)[y_2v_{22}((w_1 - w_0)[r}
\[ + (1 - \phi)s]/y_1(r + s)] - \phi(r + s)VK < 0, \]
\[ \frac{\partial}{\partial \mu}[e_y(1) - e_y(0)] = K[\partial s(1)/\partial \mu - \partial s(0)/\partial \mu]/y_1 < 0, \]
\[ \frac{\partial}{\partial \mu}[e_o(1) - e_o(0)] = (z_1 - z_o)(\rho_o y_2v_{22}/v_2 + 1)/(y_2)^2v_{22}
\[ - v_{21}K[\partial s(1)/\partial \mu - \partial s(0)/\partial \mu]/v_{22}y_2 < 0. \]

Let \( x(j) \) denote output when \( \eta_i = j. \) Then,
\[ x(j) = y_1\mu \rho_y(j) + y_2\mu \rho_o(j), \]

and,
\[ x(1) - x(0) = \mu[y_1[\rho_y(1) - \rho_y(0)] + y_2[\rho_o(1) - \rho_o(0)]]. \]

Therefore, from (36) through (38), for \( \mu \) close to 1, output is higher, unemployment rates are lower for young and old workers, and the price of capital is higher, if \( \eta_i = 1 \) than if \( \eta_i = 0. \) Thus, a sectoral disturbance is associated with high unemployment for all groups of workers, low asset prices, and low output. In terms of its predictions for the qualitative comovements of aggregates, the model is consistent with stylized facts.

If \( \eta_i = 0, \) then old workers face lower wages than they do if \( \eta_i = 1, \) since productive workers subsidize indistinguishable unproductive workers when there is a sectoral disturbance. Thus, the unemployment rate of older workers is higher and consumption of good 1 by old agents is lower when \( \eta_i = 0 \) than when \( \eta_i = 1. \) Since \( D_{12}\gamma > 0, \) the fall in consumption of good 1 decreases the marginal utility of good 2 when \( \eta_i = 0. \) Thus, the price of capital must fall in all states to induce young agents to hold the capital stock. Since \( \eta_i \) is positively serially correlated, the price of capital falls more when \( \eta_i = 0 \) than when \( \eta_i = 1. \) With the fall in the price of capital, young agents give up less good 1 to acquire the capital stock, and their
marginal utility of consumption rises. Therefore the unemployment rate of young workers must rise in equilibrium, and it rises relatively more when \( \eta_r = 0 \) than when \( \eta_r = 1 \), since the price of capital is higher when \( \eta_r = 1 \) than when \( \eta_r = 0 \).

Ignoring the sector of the economy where a constant quantity of good 1 is produced from capital, there are four sectors, producing goods 1 and 2 in green and blue. In any period in equilibrium, two of these sectors produce zero output and employ no workers. Therefore, the measure of dispersion in employment growth rates in Lilien (1982) cannot be computed for all states. As an alternative, we use the following measure:

\[
\sigma(t) = \left\{ \sum_{i=1}^{4} \left[ \text{emp}_i(t) - \text{emp}_i(t-1) \right]^2 \right\}^{1/2},
\]

where \( \text{emp}_i(t) \) is employment in sector \( i \) in period \( t \). In equilibrium, \( \sigma(t) \) depends on \( \eta_t \) and \( \eta_{t-1} \). Let \( \sigma_{ij} = \sigma(t) \) when \( \eta_t = i \) and \( \eta_{t-1} = j \). We have

\[
\sigma_{11} = 0,
\]

\[
\sigma_{10} = \left( [e_y(1) - e_y(0)]^2 + [e_o(1) - e_o(0)]^2 \right)^{1/2},
\]

\[
\sigma_{01} = \left( [e_y(0)]^2 + [e_y(1)]^2 + [e_o(0)]^2 + [e_o(1)]^2 \right)^{1/2},
\]

\[
\sigma_{00} = \left( 2[e_y(0)]^2 + 2[e_o(0)]^2 \right)^{1/2}.
\]

Note that, in the model without private information, \( \sigma_{11} = \sigma_{10} = 0 \) and \( \sigma_{01} = \sigma_{00} > 0 \) but that unemployment rates are constant. Therefore, the correlation between \( \sigma(t) \) and unemployment rates is zero with full information. With private information, the unconditional covariance of \( \sigma(t) \) and the employment rate of young workers is

\[
\text{cov}[\sigma(t), e_y(t)] = [e_y(1) - e_y(0)]\theta(1 - \theta)(1 - \theta)(1 - \phi)\sigma_{01} - \theta(1 - \phi)\sigma_{10} - [1 - \theta(1 - \phi)]\sigma_{00},
\]

and the unconditional covariance of \( \sigma(t) \) and the employment rate of old workers is

\[
\text{cov}[\sigma(t), e_o(t)] = [e_o(1) - e_o(0)]\theta(1 - \theta)(1 - \theta)(1 - \phi)\sigma_{01} - \theta(1 - \phi)\sigma_{10} - [1 - \theta(1 - \phi)]\sigma_{00}.
\]

Therefore we get, evaluating derivatives at \( \mu = 1 \):

\[
\left( \frac{\partial}{\partial \mu} \right) \text{cov}[\sigma(t), e_y(t)] = -2^{1/2}\theta(1 - \theta)[[\partial e_y(1)/\partial \mu] - [\partial e_y(0)/\partial \mu]](\rho_y^2 + \rho_o^2)^{1/2} > 0,
\]

\[
\left( \frac{\partial}{\partial \mu} \right) \text{cov}[\sigma(t), e_o(t)] = -2^{1/2}\theta(1 - \theta)[[\partial e_o(1)/\partial \mu] - [\partial e_o(0)/\partial \mu]](\rho_y^2 + \rho_o^2)^{1/2} > 0.
\]

Therefore, for \( \mu \) close to 1, employment rates for young and old workers are negatively correlated with \( \sigma(t) \). That is, unemployment rates are positively correlated with the measure of dispersion of changes in sectoral employment, as is observed empirically. Note that the model predicts that the unemployment rate of
older workers may be more or less responsive to $\sigma(t)$ than is the unemployment rate of younger workers. There are two effects which influence how unemployment rates move between states. The first is an aggregate substitution of labor for leisure in response to changes in wages. Old agents face lower wages when a sectoral disturbance occurs than when it does not, and they therefore consume more leisure in the aggregate in the first case. However, young workers face the same wage rate in all states, so that this first effect influences fluctuations in the unemployment rate of old workers only. The second effect is due to the indirect influence of fluctuations in asset prices on unemployment rates. Because of the mechanism discussed above, the fact that asset prices are lower when a sectoral disturbance occurs causes both young and old to consume more leisure in the aggregate. There is no presumption that the combined direct and indirect effects on old workers outweigh the indirect effect on young workers. However, for any parameter values, if $\phi$ is sufficiently small (i.e. if the serial correlation of disturbances is sufficiently small) then the indirect effect will be small enough so that the unemployment rate of old workers fluctuates more in response to sectoral disturbances than the unemployment rate of young workers.

Next, we will examine how the nature of fluctuations changes with changes in the parameters of the Markov process followed by the disturbances, $\eta_t$. First, the effects of a change in $\theta$, the unconditional probability that $\eta_t = 1$, are determined as follows:

$$\frac{\partial s(1)}{\partial \theta \partial \mu} = (r + s)(z_1 - z_0)v_{12}(1 - \phi)(w_1 - w_0)y_1 y_2 v_{22} < 0,$$

$$\frac{\partial s(0)}{\partial \theta \partial \mu} = (r + s)(z_1 - z_0)v_{12}(1 - \phi)(w_1 - w_0)y_1 y_2 v_{22} < 0,$$

$$\frac{\partial e_y(1)}{\partial \theta \partial \mu} = K[\partial s(1)/\partial \theta \partial \mu]/y_1 < 0,$$

$$\frac{\partial e_y(0)}{\partial \theta \partial \mu} = K[\partial s(0)/\partial \theta \partial \mu]/y_1 < 0,$$

$$\frac{\partial e_o(1)}{\partial \theta \partial \mu} = -v_{12} K[\partial s(1)/\partial \theta \partial \mu]v_{22} y_2 < 0,$$

$$\frac{\partial e_o(0)}{\partial \theta \partial \mu} = -v_{12} K[\partial s(0)/\partial \theta \partial \mu]v_{22} y_2 < 0.$$

Therefore, for $\mu$ close to 1, an increase in $\theta$ increases employment rates for all workers in all states of the world. That is, an increase in the unconditional probability of a sectoral shock (an increase in $1 - \theta$) increases the secular aggregate unemployment rate not only because a bad state of the world occurs more often, but because the unemployment rate is higher at cyclical peaks and at cyclical troughs. An increase in $\theta$ increases the probability that $\eta_{t+1} = 1$ whether $\eta_t = 0$ or $\eta_t = 1$. If $\eta_t = 1$, then the marginal utility of good 1 is relatively high for old agents, so that the price of capital increases in all states. A larger fraction of young agents is employed in all states in order to finance the purchase of capital from the old. More old agents are employed in all states because the increase in the consumption of good 1 generates an increase in the demand for good 2.

These predictions of the model are consistent with U.S. experience during the 1970s. This period is generally thought to have been one where the incidence of sectoral disturbances was relatively high. During the 1970s there was a secular
increase in unemployment rates in the United States, and the unemployment rate increased at cyclical peaks and at cyclical troughs.

Next, the effects of a change in $\phi$, the serial correlation parameter, are as follows:

$$\frac{\partial s(1)}{\partial \phi} \partial \mu = (r + s)(z_1 - z_0)\nu_{12}(1 - \theta)(w_1 - w_0)$$

$$y_1y_2\nu_{22}\{(w_1 - w_0)(r + (1 - \phi)s)\nu_{12}(r + s) - \phi(r + s) VK\}^2 < 0,$$

$$\frac{\partial s(0)}{\partial \phi} \partial \mu = -(r + s)(z_1 - z_0)\nu_{12}\theta(w_1 - w_0)$$

$$y_1y_2\nu_{22}\{(w_1 - w_0)(r + (1 - \phi)s)\nu_{12}(r + s) - \phi(r + s) VK\}^2 > 0,$$

$$\frac{\partial e_y(1)}{\partial \phi} \partial \mu = K[\partial s(1)/\partial \phi \partial \mu]y_1 < 0,$$

$$\frac{\partial e_o(0)}{\partial \phi} \partial \mu = -v_{12}K[\partial s(0)/\partial \phi \partial \mu]\nu_{22}y_2 < 0,$$

$$\frac{\partial e_o(1)}{\partial \phi} \partial \mu = -v_{12}K[\partial s(0)/\partial \phi \partial \mu]\nu_{22}y_2 > 0.$$

An increase in $\phi$ increases the degree of serial correlation in the $\eta_t$ process, and results (for $\mu$ close to 1) in an increase in employment rates for old and young agents when $\eta_t = 1$, and a decrease in employment rates when $\eta_t = 0$. Since the marginal utility of good 1 for old agents is relatively high when $\eta_t = 1$, an increase in $\phi$ increases (decreases) the expected return, in terms of marginal utility, to holding capital when $\eta_t = 1$ ($\eta_t = 0$). Therefore, the price of capital rises in state 1 and falls in state 0, and employment rates therefore increase in state 1 and decrease in state 0.

Since all variables follow a 2-state Markov process in equilibrium, the unconditional variance of any variable, $a_t$, is

$$\text{var}(a_t) = \theta(1 - \theta)[a(1) - a(0)]^2.$$ 

Therefore, unconditional variance is affected by $\phi$ only through its effect on $a(1)$ and $a(0)$. Since $[a(1) - a(0)]$ increases with an increase in $\phi$ for all variables, the variances of output, asset prices, and unemployment rates increase. This result may seem counterintuitive, since we might expect an increase in the persistence of underlying shocks to decrease volatility rather than to increase it.

5. REMARKS ON FEATURES OF THE MODEL

Several issues may require further elaboration at this point. These issues concern: first, the mechanism which causes the unemployment rates of young and old workers to move together in the model; second, the role played by private information; and third, the existence of equilibrium.

In equilibrium, the unemployment rates of young and old workers are correlated because of the consumption complementarity for old agents, which induces movements in asset prices across states of the world that in turn generate income effects for young workers. Thus, when old workers face low wages, they work less
and produce less in the aggregate, and a fall in the price of capital causes an income transfer from old agents to young agents, so that the young also work less. Two possible problems arise here. First, several results depend on the complementarity assumption. If goods 1 and 2 were substitutes for old agents, then the unemployment rates of young workers and old workers would be negatively correlated, and asset prices would be countercyclical. However, note that the correlation between generational unemployment rates could also be generated with a complementarity in production between labor supplied by the young and the old. Some form of complementarity seems to be required, but complementarity is also a key element of some prominent theories of the determinants of aggregate activity. See, for example Cooper and John (1988). Second, it may not seem plausible that the mechanism by which the unemployment rates of the young and old move together could be important for real-world aggregate fluctuations. However, it is convenient analytically in this model for the complementarity to be in consumption, and for the effects on young workers to come through movements in asset prices. In this way, the transfers between the young and the old depend only on the state variable \( \eta \) in equilibrium, and the equilibrium is therefore much easier to characterize than it would be otherwise. Introducing a capital asset in the model also introduces some dynamics, and permits the model to generate interesting predictions concerning the effects of changes in the parameters of the stochastic process governing the sectoral disturbance.

Private information is certainly not critical to the mechanism in the model by which purely sectoral shocks have aggregate implications. What is important is that information is lost as the result of a sectoral shock, and not that some agents in the economy have this information and others do not. However, private information is important here for analytical tractability. Any sectoral shifts model requires heterogeneity, and introducing heterogeneity can quickly lead to analytical difficulties. In this model, an equilibrium is simple to characterize because of the perfect pooling result. Given the setup here, perfect pooling arises in part because agents in a given generation who work with positive probability in period \( t \) all face the same period \( t \) wage. The fractions of agent types were chosen specifically to produce this feature. Private information is also important, since if a worker’s productivity were observable (perhaps imperfectly), then some identifiably productive workers would necessarily face different wages, thus adding to the heterogeneity problem.

In characterizing the equilibrium, we have assumed that all agents who face a positive wage choose an employment probability in the interval \((0, 1)\). However, in examining (17) it seems, for agents who are identifiably unproductive when they are young, that the work effort in old age (in the event of a sectoral shock) required to support the perfect pooling equilibrium may not be feasible. Therefore, one might conjecture that an equilibrium of the type characterized in Sections 3 and 4 may not exist for any parameter values and functional forms. However, consider the following example. Let \( u(c) = \ln(c) \), and \( v(c, d) = \ln(c) + \ln(d) \). Solving for an equilibrium using (18) through (25), we get

\[
\rho_y(1) = \rho_y(0) = \frac{2}{w_1 - w_0} \mu,
\]

(39)
(40) \[ \rho_o(1) = 1/(z_1 - z_0) \mu, \]
(41) \[ \rho_o(0) = (z_1 - z_0)^{-1}, \]
(42) \[ \rho_o^1(1, 1) = \rho_o^1(1, 0) = 1/(z_1 - z_0) \mu, \]
(43) \[ \rho_o^2(1, 1) = \rho_o^2(1, 0) = 0, \]
(44) \[ \rho_o^3(0, 1) = [3 \mu - (2 + [\theta(1 - \phi) + \phi])]/\mu[1 - \theta(1 - \phi) - \phi](z_1 - z_0), \]
(45) \[ \rho_o^3(0, 0) = [3 \mu - [2 + \theta(1 - \phi)]]/\mu[1 - \theta(1 - \phi)](z_1 - z_0), \]
(46) \[ \rho_o^3(0, 1) = 3/[1 - \theta(1 - \phi) - \phi](z_1 - z_0), \]
(47) \[ \rho_o^3(0, 0) = 3/[1 - \theta(1 - \phi)](z_1 - z_0), \]
(48) \[ s(0) = s(1) = y_1/(w_1 - w_0) K. \]

Here, \( \rho_o(j, k) \) is the fraction of old agents of type \( i \) employed when \( \eta_i = j \) and \( \eta_{i-1} = k \). In this example, separability in the utility function of the old helps in obtaining a closed-form solution. However, asset prices and the employment rate of the young do not fluctuate. Here, this is not a problem, since we want only to find the subset of the parameter space for which an equilibrium exists. A small perturbation to the specification will yield fluctuations as studied previously in this section, while causing only small changes in the conditions for existence of equilibrium.

From (39) through (48), an equilibrium exists if and only if the following three conditions hold.

(49) \[ \mu > [2 + \theta(1 - \phi) + \phi]/3, \]
(50) \[ z - z_0 > 3/[1 - \theta(1 - \phi) - \phi], \]
(51) \[ w_1 - w_0 > 2/\mu. \]

Therefore, from (49) through (51), for any \( \theta \in (0, 1) \) and \( \phi \in [0, 1] \), there exist sets \( S_\mu = \{ \mu; \mu > \mu^* \}, S_w = \{ (w_1 - w_0); w_1 - w_0 > w^* \}, S_z = \{ (z_1 - z_0); z_1 - z_0 > z^* \} \), such that an equilibrium exists for \( \mu \in S_\mu, (w_1 - w_0) \in S_w, \) and \( (z_1 - z_0) \in S_z \), for some \( \mu^* > 0, w^* > 0, \) and \( z^* > 0 \). That is, for any parameterization for the stochastic disturbance process, there exists a subset of the parameter space for which an equilibrium of the type studied in Sections 3 and 4 exists.

Some equilibria are displayed in Table 1. Here, we set \( \phi \), the serial correlation parameter, to 0.1, and then try to set parameters so that average unemployment rates for the young and old take on reasonable values, with \( \theta \), the unconditional probability of not obtaining a sectoral shock, varying over the range (0, 1). For large values of \( \theta \), the value put on leisure by the old relative to its value for the young (the value of \( z_1 - z_0 \) relative to \( w_1 - w_0 \)) is quite large and the employment rates of older workers \( [\rho_o(1) \) and \( \rho_o(0) \)] are quite small. It is possible to get smaller values of \( z_1 - z_0 \) and larger values of \( \rho_o(1) \) and \( \rho_o(0) \) as \( \theta \) gets smaller. Thus, it seems
difficult to generate realistic levels for unemployment rates with plausible parameter values (i.e., parameter values where the young and old value leisure similarly, and the probability of a sectoral shock is not implausibly large). This is due to the fact that one group of workers in each generation is employed with positive probability only if a sectoral shock occurs. For perfect pooling to occur in equilibrium, these workers must work with very high probability in the event of a sectoral shock. The characteristics of the model which generate this feature also aid in obtaining a simple characterization of equilibrium for analytical purposes. Since our aim here was to explain some qualitative features of business cycles, this does not seem to be a problem. We are confident that these qualitative characteristics of the model would hold in versions where analytical tractability was sacrificed to obtain more “realistic” equilibria.

6. SUMMARY AND CONCLUSIONS

In this paper, an alternative sectoral disturbance model of the business cycle was constructed. Here, the mechanism driving fluctuations is a sorting effect in the labor market. In the model, the current sectoral composition of production determines how workers are sorted according to productivity. If a sectoral disturbance occurs, there is a decrease in the value of sorting information, output decreases, and unemployment increases for young and old workers. In equilibrium, there is a positive correlation between unemployment rates and a sectoral dispersion measure. The primary effect of a sectoral disturbance is on the behavior of older workers, since it is older productive workers who effectively subsidize unproductive older workers when a sectoral disturbance occurs.

The model provides an explanation for the secular increase in unemployment rates that occurred in the 1970s. An increase in the unconditional probability of a sectoral disturbance increases the unemployment rates of all types of workers at cyclical peaks and at cyclical troughs, as was observed during this period.

University of Western Ontario, Canada
REFERENCES


