Notes on Liquidity Effects and Inflation

Steve Williamson

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The basic model comes from Alvarez, Lucas and Weber (2001). It’s a segmented markets cash-in-advance model, in which a fraction of the population trades in the bond market, and is thus on the receiving end of open market operations. I’ll use roughly the same notation as in the original paper, but this is from memory, so it may not entirely correspond to the original. Also, I’ll simplify by leaving out the velocity shocks that are in the original model (and which are important in generating money demand instability). There is a continuum of households with unit measure. A fraction $\lambda$ are traders, and a fraction $1 - \lambda$ are non-traders. Each agent has preferences given by

$$E_0 \sum_{t=0}^\infty \left( \frac{1}{1 + \rho} \right)^t \left( \frac{c_i}{1 - \alpha} \right)^{1-\alpha},$$

where $\rho > 0$ is the rate of time preference, $c_i$ is consumption, and $\alpha > 0$ is the coefficient of relative risk aversion. Here, $i = T$ denotes a trader, and $i = N$ denotes a non-trader. Each period, each household receives an endowment $y$, which it cannot consume. The endowment is sold on a competitive goods market for cash at price $P_t$. The household purchases consumption goods on the competitive goods market with cash acquired in advance. At the beginning of the period, a bond market opens in which the government and trader households participate. Also, suppose that the government has access to lump sum taxes, and taxes the traders only.

So, each period, the bond market opens, and the government issues money and one-period bonds, and levies taxes, subject to its budget constraint. Then, the goods market opens, with households purchasing goods subject to cash-in-advance constraints. Households receive the receipts from sales of their endowments, and carry assets into the next period. Traders hold a portfolio of cash and bonds, while nontraders hold only cash.

We will consider only cases where cash-in-advance constraints always bind. This makes solving the model very mechanical. Since cash-in-advance constraints bind, all cash is spent in the goods market each period, so

$$P_i y = M_i,$$

where $M_i$ is the stock of money in period $t$. The consumption of a trader is
given by

\[ P_t c_t^T = P_{t-1} y + \frac{M_t - M_{t-1}}{\lambda}, \]  

(3)
as a trader spends the receipts from the previous period’s sales of goods, plus whatever money injection occurred through open market operations in the current period. Similarly, a non-trader consumes

\[ P_t c_t^N = P_{t-1} y \]  

(4)

In order to price assets in this economy, we need to be concerned only with the consumption of the traders. Solving for a trader’s consumption from (2) and (3), we get

\[ c_t^T = \left( \frac{y}{\lambda} \right) \left( \frac{\lambda + \pi_t}{1 + \pi_t} \right). \]  

(5)

Where \( \pi_t \) denotes the inflation rate in period \( t \), which from (2) is also the growth rate of the money stock. The key feature of this model is the redistributive effect of monetary policy. Higher money growth increases the inflation rate, but it also redistributes wealth from non-traders to traders, and increases the consumption of traders.

From (1), we can determine the price of a one-period nominal bond, defined to be a bond that pays one unit of money one period hence. If \( R_t \) is the nominal interest rate, then

\[ \frac{1}{1 + R_t} = \left( \frac{1}{1 + \rho} \right) E_t \left[ \left( \frac{1}{1 + \pi_{t+1}} \right) \left( \frac{c_t^T}{c_{t+1}} \right)^{\alpha} \right], \]

or, using (5),

\[ \frac{1}{1 + R_t} = \left( \frac{1}{1 + \rho} \right) E_t \left[ \left( \frac{1}{1 + \pi_{t+1}} \right) \left( \frac{\lambda + \pi_t}{1 + \pi_t} \right)^{\alpha} \left( \frac{1 + \pi_{t+1}}{\lambda + \pi_{t+1}} \right)^{\alpha} \right]. \]  

(6)

Then, a log-linear approximation to (6) is

\[ R_t = \rho + \underbrace{E_t \pi_{t+1}}_{Fisher} + \underbrace{\frac{\alpha(1 - \lambda)}{\lambda} (E_t \pi_{t+1} - \pi_t)}_{Liquidity} \]  

(7)

In equation (7), the second term on the right-hand side is the Fisher effect (higher anticipated inflation increases the nominal interest rate one-for-one), and the second term is the liquidity effect, whereby higher current inflation is associated with a transfer to traders, which increases their consumption, and reduces the real interest rate. In a fashion similar to (7), we can approximate the real interest rate by

\[ r_t = \rho + \frac{\alpha(1 - \lambda)}{\lambda} (E_t \pi_{t+1} - \pi_t) \]  

(8)

So, note that the long-run real rate is a constant, \( \rho \), i.e. the rate of time preference.
Suppose that the central bank wants to experiment by engaging in random open market operations. In particular, suppose that $\pi_t$ is an i.i.d. random variable with mean $\bar{\pi}$. Then, from (7),

$$R_t = \rho + \bar{\pi} \left[ \frac{\lambda + \alpha(1 - \lambda)}{\lambda} \right] \pi_t + \frac{\alpha(1 - \lambda)}{\lambda} \left[ \frac{1}{\lambda + \alpha(1 - \lambda)} \right] \pi_t.$$

Therefore, the central bank observes that, when inflation goes up, the nominal interest rate goes down. So, central bankers reason, if the central bank wants to control the inflation rate, it should increase the nominal interest rate when the inflation rate is too high, and reduce the nominal interest rate when the inflation rate is too low.

Now, suppose that we are in a world where monetary policies are announced in advance (indeed, at the beginning of time), and a path for policy is a sequence $\{R_0, R_1, R_2, \ldots\}$. The question for the policy maker is then what sequence of inflation rates are implied by some arbitrary sequence of nominal interest rates? The answer comes from (7), which gives

$$\pi_{t+1} = \frac{\lambda}{\lambda + \alpha(1 - \lambda)} (R_t - \rho) + \frac{\alpha(1 - \lambda)}{\lambda + \alpha(1 - \lambda)} \pi_t.$$

Equation (9) is a difference equation that solves for the sequence $\{\pi_t\}$ given the nominal interest rate path. For example, suppose $R_t = R$ for all $t$. Then there exists a continuum of equilibria which all converge in the limit to the steady state $\pi_t = \pi$, where

$$\pi = R - \rho,$$

so in the long run the Fisher relation holds, and $r_t = \rho$.

Thus, if the central bank thinks that a lower nominal interest rate is associated with a higher inflation rate, that may be true in the short run, for the right kind of monetary experiment, but it’s not true in the long run. The Fisher relation tells us that, if we want higher inflation in the long run, this requires a higher nominal interest rate.

Indeed, suppose that the nominal interest rate is low for a long period of time at $R^*$, and then increases once and for all to $R^{**}$, as determined by the central bank. To be explicit, $R_t = R^*$ for $t = 0, 1, 2, \ldots, T$, and then $R_t = R^{**}$ for $t = T + 1, T + 2, \ldots$, where $R^* < R^{**}$. We know that there are many equilibria, and that they all converge to $\pi_t = R^{**} - \rho$ as $t \to \infty$. One interesting equilibrium to consider is one where $\pi_t = R^* - \rho$ for $t = 0, 1, 2, \ldots, T + 1$. Then, from (9),

$$\pi_t = \left\{ 1 - \left[ \frac{\alpha(1 - \lambda)}{\lambda + \alpha(1 - \lambda)} \right] \gamma^{t-T-1} \right\} R^{**} + \left[ \frac{\alpha(1 - \lambda)}{\lambda + \alpha(1 - \lambda)} \right] \gamma^{t-T-1} (R^{**} - \rho), \quad (10)$$

for $t = T + 2, T + 3, \ldots$. So, from (10), the inflation rate increases monotonically from $R^* - \rho$ and converges in the limit to $R^{**} - \rho$. Then, we can determine the path for the real interest rate, which is: $r_t = \rho$, for $t = 0, 1, 2, \ldots, T$, $r_t = R^{**} - R^* - \rho$, for $t = T + 1$, and

$$r_t = \left[ \frac{\alpha(1 - \lambda)}{\lambda + \alpha(1 - \lambda)} \right] \gamma^{t-T-1} (R^{**} - R^*) + \rho.$$
for $t = T + 2, T + 3, \ldots$. So, the real rate increases when the nominal rate goes up, and then falls monotonically, converging to $\rho$ in the limit.

So, we might characterize this policy as “tightening,” as the real interest rate went up temporarily. If we added some nonneutralities in the model which would cause monetary policy to affect aggregate output and employment, we might expect that there would be some negative effects of increasing the nominal rate in this manner. But the policy is what increases the inflation rate. We could imagine other policies, for example the nominal interest rate could be increased in steps, which would smooth the transition. But if the goal is to increase the inflation rate in the long run, an increase in the long-run nominal interest rate is necessary.

1 References