Liquidity Constraints

Liquidity constraints affect the ability of an economic agent to exchange his or her existing wealth for goods and services or for other assets. These constraints arise because of market frictions, including private information, limited commitment, transactions costs, and spatial considerations.

A BENCHMARK MODEL

To show explain what liquidity constraints are, and their implications for economic activity, it is useful to start with a simple benchmark model. Suppose a world with a continuum of households having unit mass. Time is indexed by \( t = 0, 1, 2, \ldots \), and household \( i \) has preferences given by

\[
E_0 \sum_{t=0}^{\infty} \beta^t u(c_{it}),
\]

where \( E_0 \) is the expectation operator conditional on period 0 information, \( 0 < \beta < 1 \), \( c_{it} \) is consumption, and \( u(\cdot) \) is twice continuously differentiable, strictly concave, and has the property that \( u'(0) = \infty \). Each household receives a random endowment of the perishable consumption good at the beginning of each period. That is, household \( i \) receives an endowment \( y_{it} \) in period \( t \), where \( y_{it} \) is assumed to be independent and identically distributed across households and over time. Assume that \( \underline{y} \leq y_{it} \leq \bar{y} \), where \( 0 < \underline{y} < \bar{y} \). The law of large numbers then implies that the aggregate endowment is a constant, which we will denote by \( y \). Therefore, this is an economy with no aggregate risk, but each household faces idiosyncratic risk associated with his or her endowment shocks.

Now, suppose that this economy has a complete set of markets. One market structure that gives completeness is to have contingent claims markets that open at \( t = 0 \) before households receive their period 0 endowments. All households trade on these
markets, and a particular contingent claims market involves trade in claims to the consumption good deliverable at a particular date only under a particular realization for the path of endowment shocks for all households up to that date. Given this complete set of markets, what will the equilibrium allocation of consumption across households at each date be? All households are identical at the first date, and the result will be that, in equilibrium, \( c_{it} = y \) for all \( i \) and \( t \). The complete set of contingent claims markets has provided perfect insurance for households. That is, they are able to share their risk so efficiently that each household can shed the idiosyncratic risk associated with his or her endowment shocks. Indeed, the resulting equilibrium allocation of consumption is Pareto optimal.

Models with complete markets have proved to be very useful in economics, for example in the theory of asset pricing and in business cycle modeling. However, there are many applications where it is necessary that we depart from complete markets. One such application is to what we might broadly refer to as liquidity constraints. To think about liquidity constraints we need to seriously address the frictions that will cause markets to work differently than in the complete markets case, and in some instances will cause some markets to shut down altogether. In the next sections we will explore some key departures from our benchmark model that illustrate the role of liquidity constraints.

**INCOMPLETE MARKETS - A BEWLEY MODEL**

One approach to studying market incompleteness is to simply eliminate markets in the model under consideration, without asking questions about the underlying frictions which would cause incomplete markets. Bewley (e.g. Bewley 1980?) was a pioneer in this area, and Aiyagari (1994) provides a particularly clear treatment of the implications of incomplete markets.

As an example of the Bewley approach, suppose in our benchmark model that there
is only one asset market, which a market for non-contingent bonds on which trading
occurs each period. Households can borrow and lend on this bond market. Assume
that each bond is a one-period financial instrument. In period \( t \), a bond sells for one
unit of consumption goods, and is a promise to pay \( 1 + r_{t+1} \) units of consumption
goods in period \( t + 1 \). Since there is no aggregate risk, there will exist a competitive
equilibrium where \( r_{t+1} = r \), a constant, for all \( t \).

We now need to write down the series of constraints that a household faces. The
first of these is the sequence of budget constraints

\[
c_{it} + b_{i,t+1} = y_{it} + (1 + r)b_{i,t},
\]

for \( t = 0, 1, 2, ..., \) where \( b_{it} \) is the quantity of bonds acquired by household \( i \) in period
\( t \), and \( b_{i0} = 0 \) for all \( i \). Typically in models of this type, there is also a borrowing
constraint added, which could take the form

\[
b_{it} \geq b
\]

Constraint (1) serves a technical purpose, in that it prevents a household from borrow-
ing an infinite amount so as to finance infinite consumption. Further, the constraint
will affect the household’s ability to smooth consumption over time in the face of
fluctuating income. Constraint (1) is a kind of liquidity constraint, as it potentially
prevents the household from borrowing against his or her lifetime wealth.

A competitive equilibrium will have the property that the bond market clears, that
is the net stock of bonds in the population is zero in each period. This model is
a special case of Aiyagari (1994), and so his results apply here. With Aiyagari’s
regularity conditions on \( u(\cdot) \), a competitive equilibrium will have the property that
\( r < \frac{1}{\beta} - 1 \), that is the equilibrium real interest rate is less than the rate of time
preference. This reflects a precautionary savings motive, in that households wish to
hold bonds to self-insure against the event that the have a string of bad luck, which
in this case would be a string of low endowment shocks. Over time, a household will tend to increase his or her stock of bonds when his or her endowment is large, and to decrease the stock of bonds when his or her endowment is small. What we will observe in equilibrium is some distribution of bonds across the population, and a distribution of consumption across the population. Households who have had good luck will tend to have a larger stock of bonds and higher consumption than those households who have had bad luck. The competitive equilibrium is therefore not in general Pareto optimal.

Another related application, from Bewley (1980), is to suppose that the single asset that is traded is money. For example, suppose that there is a fixed stock of money, $M$, for all $t$. Let $P_t$ denote the price level in period $t$, and consider the equilibrium where $P_t = P$, a constant, for all $t$. For the household, we can just reinterpret his or her constraints, in that $b_{i,t+1}$ is the real quantity of money carried over by the household into period $t+1$, and $b_0 = 0$ as the household’s money balances cannot fall below zero. An individual household in this setup is even more severely liquidity-constrained than was the household in the Bewley model above. This is because the household cannot borrow at all, and cannot hold interest-bearing assets. Note that, in this monetary model, a household need only use money to buy consumption goods if he or she wishes to consume more than his or her endowment. Money is essentially held for insurance purposes, so as to smooth consumption over time.

**CASH-IN-ADVANCE**

The idea for the basic cash-in-advance model seems to come from Clower (1967?), but the key initial modeling work was done mainly by Robert Lucas, with a key contribution being Lucas (1980). Most cash-in-advance applications begin with the view that the basic frictions that might give rise to cash-in-advance constrained households are unimportant, and that it is useful to proceed from the premise that money
is necessary to purchase some goods and services.

Here, suppose in our basic model that there are no assets other than money, and that the only exchanges are trades of money for goods. Assume that a household’s purchases of goods during the current period must be financed with money carried over from the last period, and also suppose that the household cannot consume his or her own endowment. Let $m_{it}$ denote the nominal money balances that household $i$ has at the beginning of period $t$, and let $P_t$ denote the price level. Then, the household’s budget constraint in period $t$ is

$$P_t c_{it} + m_{i,t+1} = P_t y_{it} + m_{it}. \quad (2)$$

The *cash-in-advance constraint* for the household is

$$P_t c_{it} \leq m_{it}. \quad (3)$$

Thus, constraint (3) is another type of liquidity constraint. In this case, the interpretation is that some class of assets, which we refer to here as money, is necessary to carry out goods market spot exchanges.

Now, suppose that there is a fixed nominal stock of money $M$. Also, suppose that in equilibrium constraint (3) binds for each household $i$. Then, since in equilibrium the entire stock of money is held by households at the beginning of period and is spent to purchase the aggregate endowment, $y$, the equilibrium price level is

$$P_t = \frac{M}{y}$$

for all $t$. Then, given (2) and (3) with equality, we have

$$m_{i,t+1} = M \frac{y_{it}}{y},$$

which then implies, from (3) with equality, that

$$c_{it} = y_{i,t-1}.$$
Therefore, in this environment, households have essentially no ability to smooth consumption relative to income, as a result of this extreme type of liquidity constraint. The distribution of consumption across households in period $t$ is determined by the distribution of income across households in the previous period.

Economists who are serious about monetary theory often treat cash-in-advance models with some disdain (see for example Wallace, ?). The problem is not that one cannot write down a model that is explicit about frictions and gives rise to cash-in-advance as an endogenous phenomenon. For example, suppose that we modify our benchmark model to permit an absence-of-double-coincidence friction of the type considered by early monetary theorists such as Jevons (?). That is, assume that households are of $N$ types, with measure $\frac{1}{N}$ households of each type, where type is indexed by $j = 1, 2, ..., N$. Type $j$ households are endowed with good $j$, and consume the good which is endowed to type $j+1$, modulo $N$. Further, suppose that a household has two members, a shopper that takes money from the household to buy goods in another market each period, and a seller who stays at home to sell the household’s endowment. There are $N$ distinct markets, and in a given period, a shopper from a household of type $j$ goes to market $j + 1$, modulo $N$, with money to buy goods, while a the seller stays behind and sells goods in market $j$. Note that this is still not enough to give us cash-in-advance, as we still need to close off the possibility of credit arrangements among households which could take place through centralized communication, as is made clear in Kocherlakota (??). Credit can be shut down by assuming that no communication is possible across markets, with buyers and sellers in a given market having no information about each other, beside the fact that sellers have identifiable goods and buyers have identifiable money balances. With competitive pricing in each of the $N$ markets, we get exactly the setup outlined above in this section, with a cash-in-advance constraint for each household. Given symmetry, there is an equilibrium where prices are the same in every market, and so
the equilibrium allocation of consumption is identical to what was specified above.

The key problem that must be addressed in cash-in-advance environments involves what happens when there are other assets than money. For example, if we permit borrowing and lending by households, why is it that goods cannot be purchased with credit? How can money be dominated in rate of return by other assets? Why is it that government bonds, for example, are not used in transactions rather than money? Many cash-in-advance applications leave these questions unanswered.

**RANDOM MATCHING**

A useful way to extend our benchmark model at this point is to expand on the explicit cash-in-advance environment above to relate it more directly to the literature on monetary search and matching. The seminal work in this literature is by Jones (1976) and Kiyotaki and Wright (1989).

Suppose as above that there is a double coincidence problem, but here assume that there is one agent in a household, and that each household is randomly matched with one other household each period. Households produce different goods, and no household can consume its own endowment. Now, for a given household, assume that the probability is $\alpha$ that they are matched with another household whose goods they consume, but that the other household does not want its goods. As well, assume that there is a probability $\gamma$ that a household is matched with another household and there is a double coincidence of wants - each household consumes the other’s goods. Suppose that $\alpha > 0$, $\gamma > 0$, and $2\alpha + \gamma < 1$. Suppose that a household in a bilateral match has no information about the other household, except that it can observe its quantity of money balances and its endowment. Thus, exchange can only involve bilateral exchanges of goods and money.

Now, suppose that household $i$ and household $k$ are matched. There is probability $\alpha$ that household $i$ is a seller and $k$ is a buyer. In this case, we have $c_{it} = 0$, $c_{kt} = y_{it}$,
and
\[ m_{i,t+1} = m_{i,t} + m(y_{it}, m_{it}, m_{kt}), \quad m_{k,t+1} = m_{k,t} - m(y_{it}, m_{it}, m_{kt}), \]
where \( m(y_{it}, m_{it}, m_{kt}) \) is the quantity of money exchanged for the \( y_{it} \) units of goods given up by the seller when the seller has \( m_{it} \) units of money and the buyer has \( m_{kt} \) units of money. As money balances must be non-negative, we have
\[ -m_{it} \leq m(y_{it}, m_{it}, m_{kt}) \leq m_{kt} \tag{4} \]
and these constraints are essentially liquidity constraints. Similarly, with probability \( \alpha \), household \( i \) is the buyer and \( k \) is the seller, in which case \( c_{it} = y_{kt}, c_{kt} = 0 \), and
\[ m_{i,t+1} = m_{i,t} - m(y_{kt}, m_{kt}, m_{it}), \quad m_{k,t+1} = m_{k,t} + m(y_{kt}, m_{kt}, m_{it}), \]
with
\[ -m_{kt} \leq m(y_{kt}, m_{kt}, m_{it}) \leq m_{it} \tag{5} \]
Finally, with probability \( \gamma \) there is a double coincidence, and household \( i \) and \( k \) exchange goods, so that \( c_{it} = y_{kt}, c_{kt} = y_{it} \), and
\[ m_{i,t+1} = m_{i,t} + b(y_{it}, y_{kt}, m_{it}, m_{kt}), \quad m_{k,t+1} = m_{k,t} - b(y_{it}, y_{kt}, m_{it}, m_{kt}), \]
where
\[ -m_{it} \leq b(y_{it}, y_{kt}, m_{it}, m_{kt}) \leq m_{kt} \tag{6} \]
Here, \( b(y_{it}, y_{kt}, m_{it}, m_{kt}) \) is the quantity of money passed from household \( k \) to household, which depends on the money balances and endowments of each household.

Note that this environment will give a clear sense in which money improves the equilibrium allocation. If money is not valued, then households can trade only when there is a double coincidence of wants, and this could severely limit exchange possibilities. In principle, the constraints (4), (5), and (6) will matter for the equilibrium allocation in important ways. However, the model as we have laid it out is quite
intractable. It is possible to use a bargaining approach, as for example in Trejos and Wright (1995) or Shi (1995), to determine the how much money is transferred in each type of match, but they key problem is in tracking the distribution of money balances in the population over time.

In some of the monetary search and matching literature, tractability is achieved through assuming that money and goods are indivisible (Kiyotaki and Wright 1989) or that money is indivisible and goods are divisible (Trejos and Wright 1995, Shi 1995), and that there is an inventory constraint on money holdings. If a household can hold only one unit of money or nothing and money is never disposed of, then the quantity of money outstanding tells us how many households have it and how much, and how many do not have it. Models with indivisible money yield some insights, but they are extremely awkward for dealing with some types of policy questions, such as those involving money growth and the effects of inflation. Some recent progress in the development of tractable search models of divisible money was achieved by Lagos and Wright (1995), who use a quasilinear utility setup with labor supply and alternating periods of centralized meeting and search. This type of model yields a result where, in the periods when centralized meeting takes place, economic agents optimally redistribute money among themselves in such a way that the distribution of money balances becomes degenerate. Recent research using this type of model (e.g. Williamson 1995?, Berentsen, Camera, and Waller 1995, ??) has been quite productive.

**PRIVATE INFORMATION AND LIMITED COMMITMENT**

As an alternative to shutting down markets in an ad-hoc fashion, imposing borrowing constraints, assuming cash-in-advance constraints, or making extreme informational assumptions that shut down all trade except monetary exchange, there are available approaches to facing frictions head-on that lead to incomplete insurance and
imperfect credit. These approaches involve economies with private information and limited commitment.