Notes on Prices

Stephen Williamson
Washington University in St. Louis

April 10, 2011
I think that one could fill in the details of production, consumption, monetary exchange, and central banking in this economy and get much the same thing, but I’ll lay out the argument in a very stark way. Suppose there are two goods, indexed by \( i = 1, 2 \), with prices in units of money \( P_t^i \), for \( i = 1, 2 \). Assume that there are random movements in relative prices, so that

\[
P_t^2 = \Omega_t P_t^1,
\]

where \( \Omega_t \) is a stochastic process. Also suppose that

\[
(P_t^1)^\alpha (P_t^2)^{1-\alpha} = \Gamma_t M_t,
\]

where \( \Gamma_t \) is another stochastic process, with \( \Gamma_t \) independent of \( \Omega_t \). Here, \( \Gamma_t \) captures errors in monetary policy and unpredictable movements in the velocity of money. The expression on the left-hand side of (2) is a geometric weighted average of the two prices - a price index - with \( 0 < \alpha < 1 \).

Now, let lower case variables denote natural logarithms, and suppose that the central bank controls \( m_t \) to minimize the variance of \( p_t^1 - p^* \), where \( p^* \) denotes a price level target. Then,

\[
p_t^1 = p^* + \gamma_t - E[\gamma_t | \Phi_t] - (1-\alpha) \{\omega_t - E[\omega_t | \Phi_t]\},
\]

\[
p_t^2 = p^* + \gamma_t - E[\gamma_t | \Phi_t] + \alpha \omega_t + (1-\alpha) E[\omega_t | \Phi_t],
\]

where \( \Phi_t \) is the information set available to the central bank at the beginning of period \( t \). Further, the price index is given by

\[
\alpha p_t^1 + (1-\alpha)p_t^2 = p^* + \{\gamma_t - E[\gamma_t | \Phi_t]\} + (1-\alpha) E[\omega_t | \Phi_t]
\]

Now, to simplify, suppose further that

\[
\omega_t = \rho_1 \omega_{t-1} + \varepsilon_t^1,
\]

\[
\gamma_t = \rho_2 \gamma_{t-1} + \varepsilon_t^2,
\]

where \( 0 < \rho_i < 1 \) for \( i = 1, 2 \), and \( \varepsilon_t^1 \) and \( \varepsilon_t^2 \) are mutually independent innovations with means \( \mu_i \), for \( i = 1, 2 \). Then, (3)-(5) give

\[
p_t^1 = p^* + \varepsilon_t^1 - (1-\alpha)\varepsilon_t^1,
\]

\[
p_t^2 = p^* + \varepsilon_t^2 + \alpha \omega_t + (1-\alpha) (\rho_1 \omega_{t-1} + \mu_1),
\]

\[
\alpha p_t^1 + (1-\alpha)p_t^2 = p^* + \varepsilon_t^2 + (1-\alpha) (\rho_1 \omega_{t-1} + \mu_1)
\]

Now, the central bank looks at the data and says: Yes, we are correct in focusing our attention on the price of good 1. From equations (6)-(8), the price of good 1 is smooth relative to the price of good 2, and since the relative price movements are transient, deviations of the good 2 price from the target are also transient. Further, since movements in the price of good 1 represent only the innovations in the underlying stochastic processes, good 1 is a wonderful forecasting tool for the future price index. Of course, the central bank could have chosen the price of
good 2 to target, and could then make exactly the same argument to justify that choice. Ultimately, if the central bank determines that what matters for welfare is the price index, then it should ignore the relative price shocks. But this does not mean focusing on some subset of the prices, as this optimal behavior will yield

\[
\begin{align*}
    p_t^1 &= p^* + \varepsilon_t^2 - (1 - \alpha)\omega_t \\
    p_t^2 &= p^* + \varepsilon_t^2 + \alpha \omega_t \\
    \alpha p_t^1 + (1 - \alpha)p_t^2 &= p^* + \varepsilon_t^2
\end{align*}
\]