Liquidity, Monetary Policy, and the Financial Crisis: A New Monetarist Approach

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A model of public and private liquidity is constructed that integrates financial intermediation theory with a New Monetarist monetary framework. Key features of the model are non-passive fiscal policy and costs of operating a currency system, which imply that an optimal policy deviates from the Friedman rule. A liquidity trap can exist in equilibrium away from the Friedman rule, and there exists a permanent non-neutrality of money, driven by an illiquidity effect. Financial frictions can produce a financial-crisis phenomenon, that can be mitigated by conventional open market operations working in an unconventional manner. Private asset purchases by the central bank are either irrelevant or they reallocate credit and redistribute income.

JEL: E3, E4, E5

Keywords: Liqudity, Monetary Policy, Financial Crisis, New Monetarism

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1 Introduction

Liquidity is a class of assets that is useful in exchange. Some of these liquid assets are government liabilities - Federal Reserve notes, Treasury bills, and Treasury notes. Some are private liabilities - the deposit liabilities of banks and some asset-backed securities, for example. Conventional wisdom holds that the main role of a central bank is to manage public liquidity in a manner that controls inflation, and enhances the provision of private liquidity and credit. However, the mechanism by which central bank actions work to perform this role still appears to be poorly understood, as has been highlighted by the financial crisis and the ensuing policy responses.

The purpose of this paper is to build a model of public and private liquidity that captures the features of modern economies essential to the analysis of monetary policy, and that can be used to shed light on some current monetary policy problems and questions. What is a liquid asset, and what roles do privately-provided and publicly-provided liquid assets play in exchange? In a model where we capture these roles for liquid assets, what are the implications for monetary policy? Under what circumstances can a liquidity trap occur? How does monetary policy work when there is a positive quantity of excess reserves held by banks? What does a financial crisis do to the supply of private liquidity, and what should monetary policy do in response?

The model constructed here brings together and builds on two branches of the literature. The first branch involves research on financial intermediation and macroeconomic credit frictions, including models of costly state verification and delegated monitoring, building on Townsend (1979) and Diamond (1984) (e.g. Williamson 1986, 1987, Bernanke and Gertler 1989, Bernanke, Gertler and Gilchrist 1999), and models derived from the risk-sharing banking framework of Diamond-Dybvig (1983). The second branch includes a class of explicit models of money, liquidity, and asset exchange - "New Monetarist Economics" in the language of Williamson and Wright (2010, 2011). Important contributions to this literature include Kareken and Wallace (1980), Kiyotaki and Wright (1989), and particularly Lagos and Wright (2005). Key research relating to asset exchange and pricing that has a bearing on what we will do here are papers by Lagos (2008), Lester, Postlewaite, and Wright (2009), and Lagos and Rocheteau (2008).

The issues we study are also related to some recent papers which use different modeling approaches. Kiyotaki and Moore (2008) examine credit frictions arising from exogenous liquidity constraints, while Gertler and Kiradi (2010) and Gertler and Kiyotaki (2011) look at non-monetary models with limited commitment frictions. Curdia and Woodford (2010) extend existing New Keynesian sticky price models to include financial frictions. The value-added in our paper relative to this collection of work is the explicit use of received intermediation theory and monetary theory, an explicit treatment of monetary policy that identifies assets and liabilities in the model with the key entries in real-world central bank balance sheets, and the incorporation of retail currency transactions, among other things.

The basic model builds on Lagos and Wright (2005) and Rocheteau and Wright (2005). As in those models, quasilinear preferences are useful for analytical tractability. To understand the basic forces at work in the model, we deal first with a baseline environment where there are two government-supplied assets: currency and nominal government bonds. Currency can be issued only by the central bank, and bonds are accounts with the fiscal authority. In this environment, there is a role for banking similar to what Diamond and Dybvig (1983) study, in that banks permit liquidity to be efficiently allocated when individual demands for liquidity are uncertain. In contrast to a Diamond-Dybvig environment though, bank liabilities are traded in equilibrium and rates of return on assets are endogenous.

Two features of the model will be important for monetary policy implications. First, in general the actions of the central bank are constrained, in reality, by what the fiscal authority does. Second, optimal monetary policy depends in an important way, as in part argued in Sanches and Williamson (2010), on the costs of operating a currency system. These costs include direct costs, such as the costs of designing the currency to thwart counterfeiters, printing currency, and destroying worn-out notes; and indirect social costs, such as theft, trade in illegal goods and services, and tax evasion.

In the benchmark economy where there is only public liquidity (currency and government bonds), the monetary authority sets policy with taxes responding passively, and there are no costs of operating the currency system. Here, an equilibrium can be one of four types: a liquidity trap equilibrium, an equilibrium with plentiful interest-bearing assets, an equilibrium with scarce interest-bearing assets, or a Friedman rule equilibrium.

In a liquidity trap equilibrium the nominal interest rate is zero, excess reserves are held by banks, and open market operations are irrelevant (at the margin) for equilibrium quantities and prices. A novelty here is that the liquidity trap equilibrium is not associated with the Friedman rule; indeed, it can exist for essentially any long-run money growth rate. A liquidity trap arises if total liquid assets (public and private) are sufficiently scarce, and currency is sufficiently plentiful relative to other assets. This helps in understanding current observations with regard to interest rates and monetary policy in the United States.

In an equilibrium with plentiful interest-bearing assets, trading is efficient in non-currency transactions, and thus there is no liquidity premium associated with interest-bearing assets. Open market operations have standard effects in this equilibrium, in that a one-time permanent open market purchase of nominal government bonds serves to increase the price level in proportion to the money injection, and the real interest rate is unaffected. Things are quite different, though, in the equilibrium with scarce interest-bearing assets. Here, there is a liquidity premium on interest-bearing assets, reflected in a real interest rate that is less than the rate of time preference. A one-time permanent open market purchase, while it results in a proportionate increase in the price level, with no effect on the real stock of currency, acts to make public liquidity more scarce, and the real interest rate falls. There is an *illiquidity* effect of an open market purchase, i.e. a nonneutrality of money which is related to results in Lagos (2008) and Lagos and Rocheteau (2008), concerning asset scarcity and liquidity premia.

It is straightforward to analyze a regime where interest is paid on bank reserves, so that we can gain some insight into how central bank policy works in the operating environment faced by the Fed since late 2008. If excess reserves are held in equilibrium, then open market operations are irrelevant at the margin, much like in the liquidity trap equilibrium, but with a positive nominal interest rate. Monetary policy works in this regime through changes in the interest rate on reserves, which essentially determines all short-term market interest rates.

To include non-passive fiscal policy we look at a regime where the fiscal authority fixes the real deficit forever, and the central bank must treat this as a constraint. As well, we capture the costs of operating a currency system in the simplest possible way, by supposing that a fixed fraction of currency transactions are deemed illegal and therefore socially useless. Given this, we characterize an optimal monetary policy, which in general deviates from the Friedman rule. In the model, inflation is beneficial as it taxes socially-undesirable currency transactions, but it is also costly in that it taxes socially-desirable currency transactions and can act to indirectly disrupt exchange that does not involve currency.

The final step in our analysis is to include the production of private liquid assets, by employing a costly-state-verification delegated-monitoring credit sector similar to that in Williamson (1987). We can then study how financial frictions contributed to the financial crisis, and explore the role of monetary policy in the crisis. In response to a financial crisis, conventional monetary policy should act to mitigate the liquid-asset scarcity, but since the scarcity is in terms of interest-bearing assets, this mitigation typically involves *increasing* the real interest rate, which may be accomplished through an open market sale of government bonds. In contrast, the Great-Depression-era financial crisis described by Friedman and Schwartz (1963) involved a different type of liquidity scarcity - essentially a shortage of currency - which is appropriately mitigated through open market purchases by the central bank of interest-bearing government liabilities.

Finally, we use the model to study private asset purchases by the central bank, of the type carried out by the Fed in its first "quantitative easing" program. At best, central bank purchases of private assets have no effects on prices or quantities in the model, but they increase the nominal stock of outside money and add a layer of redundant financial intermediation. However, if private asset purchases by the central bank are made on better terms than the private sector is offering, this will act to reallocate credit and redistribute wealth, with no obvious net benefits.

The remainder of the paper is organized as follows. The second section is a description of the baseline model, and Section 3 contains the construction and analysis of an equilibrium with passive fiscal policy. Then, in Sections 4 through 6, non-passive fiscal policy, costs associated with currency, and private liquidity, are added to the model in succession. The last section is a conclusion.

2 The Baseline Model

The baseline model builds on Lagos-Wright (2005) with heterogeneity among economic agents similar to Rocheteau-Wright (2005), and an information structure related to Sanches and Williamson (2010). The financial intermediation sector shares features with Diamond and Dybvig (1983) and Berentsen-Camera-Waller (2007), though in later sections we will add elements that will yield an intermediary structure resembling Williamson (1987). Time is indexed by $t = 0, 1, 2, ...,$ and there are two subperiods within each period. In the first and second subperiods, there is trading in a *centralized market* (CM) and a *decentralized market* (DM), respectively.

2.1 Private Economic Agents

The population consists of two types of economic agents: buyers and sellers. There is a continuum of buyers with mass one, and each buyer has preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t [-H_t + u(x_t)],$$

Here, $0 < \beta < 1$, H_t denotes the difference between labor supply and consumption in the

CM, x_t is consumption in the DM, and $u(\cdot)$ is a strictly increasing, strictly concave, and twice continuously differentiable function with $u(0) = 0$, $u'(0) = \infty$, $u'(\infty) = 0$, $-x\frac{u''(x)}{u'(x)} < 1$ for all $x \geq 0$, and with the property that there exists some $\hat{x} > 0$ such that $u(\hat{x}) - \hat{x} = 0$. Define x^* by $u'(x^*) = 1$. There is a continuum of sellers, also with unit mass, and each seller has preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t],$$

where X_t is consumption in the CM and h_t is labor supply in the DM. The production technology potentially available to buyers and sellers allows the production of one unit of the perishable consumption good for each unit of labor supply. Buyers can produce only in the CM, and sellers only in the DM, so there is a role for intertemporal exchange.

In the DM, each buyer is matched at random with a seller. The seller in a match is not able to observe the buyer's history, and the seller will never have an opportunity to signal default on a credit arrangement, so the seller will not accept a personal IOU in exchange for goods. A fraction ρ of DM bilateral meetings is *not monitored*, in the sense that, if the buyer wants to acquire goods from the seller, he or she must have a claim that can be exchanged for goods in the future, where the claim is somehow documented in an object that the buyer carries. In this model, the only physical object that potentially has these properties (when it is valued in equilibrium) is currency issued by the government. We assume that it is costless for the government to issue perfectly durable and divisible currency that is not counterfeitable, and that private circulating notes are not issued, either because the government prohibits this, or because it is unprofitable.[1]. For now we will assume that there are no costs (direct costs or indirect social costs) to operating a currency system, but we will relax this later in the paper. A fraction $1 - \rho$ of buyers and sellers are in *monitored* meetings in the DM. In these meetings, though a credit transaction cannot take place between the buyer and the seller, a communication technology is costlessly available which permits the buyer to transfer ownership of a claim on a financial intermediary to the seller. Assume that, when a buyer meets a seller, the buyer makes a take-it-or-leave-it offer of assets in exchange for goods.[2]

[1] There might be concern that we are not addressing explicitly why private circulating notes are not issued. While some countries have explicit legal restrictions prohibiting the issue of objects resembling government-issued currency (Canada banned private currency issue in 1944, for example), according to Schuler (2001), the United States no longer does. Explaining why U.S. banks do not issue close substitutes for currency is an unanswered research question, outside the scope of this paper, and potentially irrelevant for the issues addressed here.

[2] There are many ways to split the surplus from trade, including Nash bargaining (Lagos-Wright 2005), competitive search (Rocheteau-Wright 2005), or competitive pricing (Rocheteau -Wright 2005 or Andolfatto 2009, for example). Here, given that the seller's utility is linear in labor supply, take-it-or-leave-it offers by the buyer are equivalent to competitive

In the CM, all sellers, buyers, and the government meet in a centralized Walrasian market, where there is lack of recordkeeping, except for records of the ownership of claims to accounts with financial intermediaries and the government. Finally, after all production and consumption decisions are made during the CM, buyers learn whether they will be in non-monitored or monitored meetings with sellers in the following DM, and this information is public. This will give rise to a Diamond-Dybvig (1983) risk-sharing role for financial intermediaries.

2.2 Government

We will first deal with the government as a consolidated entity; later we will consider issues of how we can separate monetary policy from fiscal policy. First, assume that the government has the power to levy lump-sum taxes on buyers in the CM, with τ_t denoting the tax per buyer in units of goods. As well, the government has M_t units of currency outstanding in period t, and issues B_t one-period nominal bonds held by the private sector. All government asset transactions take place in the CM. A government bond is an account balance held with the government which sells in the CM of period t for one unit of money, and pays off q_{t+1} units of money in the CM of period $t+1$. Assume that the government collects taxes before the Walrasian market opens in the CM. Then, letting ϕ_t denote the price of money in terms of goods in the CM Walrasian market, the consolidated government budget constraint is

$$\phi_t(M_t + B_t) + \tau_t = \phi_t(M_{t-1} + q_t B_{t-1}). \tag{1}$$

Equation (1) states that the real value of the government's net outstanding liabilities at the end of the CM in period t, plus tax revenue collected, must equal the government's net outstanding liabilities at the beginning of the CM, for $t = 1, 2, ..., \infty$. Assume that the government starts period 0 with no outstanding liabilities, so

$$\phi_0(M_0 + B_0) + \tau_0 = 0, \tag{2}$$

i.e. private agents are endowed with no outside assets at the first date.

3 Equilibrium With Passive Fiscal Policy

Before enriching the model in a way that allows us to address some particular policy issues, we will consider a simpler benchmark structure that will allow us to highlight some key mechanisms at work in the model. In the benchmark case we adopt an approach that is standard

pricing. Take-it-or-leave-it lends tractability to the problem, and avoids distractions associated with determining how the surplus from trade is split. One could argue that bargaining is not central to the issues we wish to address here.

in the monetary economics literature, which is to treat policy as being driven by the monetary authority, with the fiscal authority levying the taxes that are required to support that policy.

In this model, arbitrage implies that currency is in general dominated in rate of return by all other assets. Letting r_{t+1} denote the gross real interest rate on government debt, i.e. $r_{t+1} = \frac{\phi_{t+1} q_{t+1}}{\phi_t}$, and noting that quasilinear utility implies that the real rate of interest cannot exceed the rate of time preference, in equilibrium we have

$$\frac{\phi_{t+1}}{\phi_t} \leq r_{t+1} \leq \frac{1}{\beta}. \tag{3}$$

3.1 Banks

Now, in the CM there is a Diamond-Dybvig (1983) role for banks that can insure against the need for liquid assets in different types of transactions. A bank can be run by any individual. Banks form in the CM, before buyers know whether they will be in a non-monitored or monitored meeting in the subsequent DM, and they dissolve in the CM of the subsequent period, when they are replaced by a new set of banks.

In equilibrium a bank offers a deposit contract that maximizes the expected utility of each of its identical depositors, and earns zero profits. The depositors are buyers who work in the CM and then make deposits in the form of goods when the bank forms. The bank acquires enough deposits from each depositor to purchase m units of currency (in real terms) and a units (also in real terms) of interest-bearing assets. In the benchmark model, interest-bearing assets are government bonds, but later we will modify the model to include privately-created interest-bearing assets.

When depositors learn their types, at the end of the CM, each depositor who will be in a non-monitored meeting in the DM withdraws $\frac{m'}{\rho}$ units of currency. Depositors in monitored meetings each receive the right to trade away deposit claims on $\frac{m-m'+a-a'}{1-\rho}$ units of the bank's original assets. After the claims (in the form of deposits and currency) of the original depositors are traded away in the DM, the original depositors still have claims on a' interest-bearing assets. Without loss of generality (as this will not matter for the expected utility of the depositors), assume the bank assigns these claims to the monitored depositors, who then receive the returns to these assets in the next CM. Note that we are assuming that all currency held by the bank is ultimately traded away by the depositors in equilibrium.[3] Thus, an equilibrium deposit contract

[3] When the nominal interest rate is zero, there can be equilbria where currency is willingly held from one period to the next, but we do not lose anything from ignoring these equilibria.

(m, a, m', a') solves

$$\max_{m,a,m',a'} \begin{pmatrix} -m - a + \rho u\left(\beta \frac{\phi_{t+1}}{\phi_t} \frac{m'}{\rho}\right) \\ +(1-\rho)\left\{u\left[\beta r_{t+1}\left(\frac{a-a'}{1-\rho}\right) + \beta \frac{\phi_{t+1}}{\phi_t} \frac{(m-m')}{(1-\rho)}\right] + \beta r_{t+1}\left(\frac{a'}{1-\rho}\right)\right\} \end{pmatrix} \tag{4}$$

subject to $m \geq 0$, $a \geq 0$, $0 \leq m' \leq m$, and $0 \leq a' \leq a$. In (4), given take-it-or-leave-it offers by buyers in DM meetings, each non-monitored depositor receives $\beta \frac{\phi_{t+1}}{\phi_t} \frac{m'}{\rho}$ goods from the seller they meet in exchange for their currency, while each monitored depositor receives $\beta r_{t+1}\left(\frac{a-a'}{1-\rho}\right) + \beta \frac{\phi_{t+1}}{\phi_t} \frac{(m-m')}{(1-\rho)}$ goods in exchange for his or her deposit claims. Given the restrictions on equilibrium rates of return from the arbitrage conditions (3), the solution to problem (4) is:

1. [*Liquidity trap case*] If $\frac{\phi_{t+1}}{\phi_t} = r_{t+1} < \frac{1}{\beta}$, then $a' = 0$, $m' = \rho(a+m)$, and $m \geq \frac{\rho a}{1-\rho}$, where $a + m$ satisfies
$$\beta r_{t+1} u'\left[\beta r_{t+1}(a+m)\right] = 1, \tag{5}$$
2. [*Plentiful interest-bearing assets case*] If $\frac{\phi_{t+1}}{\phi_t} < r_{t+1} = \frac{1}{\beta}$, then $m' = m$, $a \in [(1-\rho)x^*, \infty)$, $a' = a - x^*$, and m solves
$$\left(\frac{\beta \phi_{t+1}}{\phi_t}\right) u'\left(\frac{\beta \phi_{t+1}}{\phi_t} \frac{m}{\rho}\right) = 1, \tag{6}$$
3. [*Scarce interest-bearing assets case*] If $\frac{\phi_{t+1}}{\phi_t} < r_{t+1} < \frac{1}{\beta}$, then $m' = m$, $a' = 0$, and m and a solve, respectively, (6) and
$$\beta r_{t+1} u'\left(\frac{\beta r_{t+1} a}{1-\rho}\right) = 1. \tag{7}$$
4. [*Friedman rule case*] If $\frac{\phi_{t+1}}{\phi_t} = r_{t+1} = \frac{1}{\beta}$, then $m \geq \rho x^*$, $m' = \rho x^*$, $a' = a + m - x^*$, and $a + m \geq x^*$.

In case 1, the rates of return on money and other assets are equal (the nominal interest rate is zero), so monitored and non-monitored depositors consume the same amount in the DM. The bank must hold enough currency to finance the consumption of non-monitored depositors, but the bank is otherwise indifferent about the composition of assets in its portfolio. Indeed, in this case the bank may choose to hold outside money as reserves until the next CM when the bank is liquidated. In case 2 the rate of return on interest-bearing assets is equal to the rate of time preference. In this case, exchange is efficient for monitored depositors (all these agents buy x^* in the DM) and the bank is willing to acquire an unlimited quantity of interest-bearing assets (in excess of what is required for monitored depositors to purchase x^*) so that monitored depositors can hold them until the next CM, without trading them in the DM. In case 3, money is dominated in rate of return by other assets, and these other assets have a rate of return less than the rate of time preference. In this case, a bank's deposit contract stipulates that all of the currency held by the bank is withdrawn by non-monitored depositors and spent in the DM, and all the remaining deposits in the bank (which are backed by assets other than currency) are traded in the DM. Finally, in case 4, where all rates of return are equal to the rate of time preference, monitored and nonmonitored exchange in all DM meetings is efficient, and the bank is willing to acquire an unlimited quantity of all assets to carry over until the next CM.

What does the bank accomplish here in the way of risk sharing? Consider what would happen if there were no banks. Then, each buyer would leave the CM with a portfolio of currency and government bonds. Then, if the buyer was in a non-monitored trade, the government bonds would be of no use, since they would not be acceptable in trade, and if the buyer was in a monitored trade, currency would be used inefficiently, since it would in general be dominated in return by government bonds. The bank essentially permits the efficient allocation of liquidity in transactions. There is a Diamond-Dybvig (1983) insurance role for banking, but in contrast to the Diamond-Dybvig model, liquidity and the rates of return on assets are endogenous here, and bank deposits are tradeable.

3.2 Government Policy

It is typical in the monetary theory literature to treat fiscal policy as being purely passive. For example, in Lagos-Wright (2005), which is representative in this respect, the authors analyze what they consider a monetary policy experiment. This involves examining the effects of changing the money growth rate, assuming that the path of lump-sum taxes changes passively to support different paths for the nominal money stock. We follow a similar approach in this section, as a benchmark case so we can show later how the fiscal policy regime matters for monetary policy.

Now, to limit the class of monetary policies so that our analysis is productive, suppose that the monetary authority commits to a policy such that the total stock of nominal government liabilities grows at a constant gross rate μ, and the ratio of currency to the total nominal government debt is a constant, δ. That is,

$$M_t = \delta(M_t + B_t). \tag{8}$$

Here, B_t denotes the bonds of the consolidated government (the fiscal and monetary authorities combined) held by the private sector. We allow for equilibria where the nominal interest rate is zero, in which case some of the stock of outside money (M_t) may be held by banks as reserves. Note that, in principle, $\delta \in (-\infty, \infty)$ is admissible, and if $\delta < 0$ ($\delta > 0$) then the consolidated government is a net creditor (debtor).

Given the class of monetary policies under consideration, the arbitrage conditions (3), and the government budget constraints (1) and (2), lump sum taxes are passively determined by

$$\tau_t = -\frac{\phi_t M_t}{\delta}\left(1-\frac{1}{\mu}\right) + \frac{\phi_t M_{t-1}(1-\delta)}{\delta}(\mu r_t - 1), \tag{9}$$

$$\tau_0 = -\frac{\phi_0 M_0}{\delta}. \tag{10}$$

Now, in equation (9), the first term on the right-hand side is the negative of the proceeds from the increase in the stock of total government liabilities in period t, which reduces taxation, and the second term is the real value of the net interest on government liabilities. Equation (10) determines the real transfer that goes to the private sector as the proceeds of the initial issue of government liabilities at $t = 0$.

3.3 Equilibrium

We will confine attention to stationary equilibria where real quantities are constant over time. This then implies that the gross real return on currency is $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$. Then, we define a stationary equilibrium as follows.

Definition 1 *Given a monetary policy* (μ, δ)*, a stationary equilibrium with passive fiscal policy consists of real quantities of currency* m *and interest-bearing assets* a*, a tax* τ *for periods* $t = 1, 2, ...,$ *an initial tax* τ_0*, and a gross real interest rate* r*, such that (i)* m *and* a *solve (4) when* $\frac{\phi_{t+1}}{\phi_t} = \frac{1}{\mu}$ *and* $r_{t+1} = r$*; (ii) asset markets clear*

$$a = m\left(\frac{1}{\delta} - 1\right); \tag{11}$$

and (iii) the government budget constraints (9) and (10) hold, or

$$\tau = -\frac{m}{\delta}\left(1-\frac{1}{\mu}\right) + m\left(\frac{1}{\delta}-1\right)\left(r - \frac{1}{\mu}\right), \tag{12}$$

$$\tau_0 = -\frac{m}{\delta}. \tag{13}$$

For existence of this equilibrium, it is necessary that $\mu \geq \beta$, i.e. the nominal interest rate must be non-negative. Our next step is to characterize equilibria, but how the model behaves depends critically on the relative returns on currency and other assets, as we saw already in the solution to the bank's problem. There are four different cases to consider, which are considered in turn in the next four subsections.

3.3.1 Liquidity Trap Equilibrium

In the liquidity trap case, $\frac{1}{\mu} = r < \frac{1}{\beta}$, so the real rates of return on currency and interest-bearing assets are equal (the nominal interest rate is zero). However interest-bearing assets are scarce, as reflected in a real rate of return less than the rate of time preference.

Here, the solution to the bank's problem, (5) gives

$$\frac{\beta}{\mu}u'\left(\frac{\beta}{\mu}(a+m)\right) = 1,$$

and this equation and (11) give

$$\frac{\beta}{\mu}u'\left(\frac{\beta m}{\mu\delta}\right) = 1, \tag{14}$$

which solves for m, the real quantity of outside money acquired by banks in the CM. Now, in this equilibrium non-monitored depositors withdraw $\frac{\rho m}{\delta}$ units of currency from banks at the end of the CM, leaving $\frac{m(\delta-\rho)}{\delta}$ in outside money that is held as reserves by banks until the next CM. The necessary and sufficient condition for this equilibrium to exist is that reserves be nonnegative, i.e. $\delta \geq \rho$, so this equilibrium exists in the set

$$\{(\delta,\mu) : \mu > \beta,\ \delta \geq \rho\} \tag{15}$$

This equilibrium is a liquidity-trap equilibrium, since it has the property that the nominal interest rate is zero, and from equation (14), the real stock of outside money is proportional to δ. A change in δ, essentially a one-time open market operation, is irrelevant - it leaves all prices and quantities unaffected. For example, an increase in δ implies that a one-time open market injection of outside money is held as bank reserves forever, and there is no effect on the price level.

A key result here, which is new in the literature, is that this equilibrium is not a Friedman rule equilibrium, in spite of the fact that the nominal interest rate is zero. In most monetary models, if the economy is stationary with no aggregate shocks, the nominal interest rate is zero forever when the central bank runs the Friedman rule, implemented for example if the money supply grows at minus the rate of time preference. Here, the central bank can achieve a liquidity trap equilibrium with any money growth factor $\mu > \beta$ given judicious choice of δ, as determined by (15).

The government is not powerless in a liquidity trap equilibrium. From equation (14), changing δ has no real (or nominal) effects, but changing μ, the gross growth rate in nominal government liabilities, matters. Indeed, an increase in μ (equal to the gross inflation rate in equilibrium) results in a decrease in m, and causes consumption to fall in the DM for all buyers. Further, an increase in μ lowers the real interest rate r.

3.3.2 Equilibrium With Plentiful Interest-Bearing Assets

In this equilibrium, $\frac{1}{\mu} < r = \frac{1}{\beta}$. Here, the nominal interest rate is positive, so that currency is scarce relative to other assets. However $r = \frac{1}{\beta}$, so that government bonds are not scarce, i.e. there is no liquidity premium on bonds.

Now, from the bank's problem, when $\frac{1}{\mu} < r = \frac{1}{\beta}$, m solves (6), or

$$\frac{\beta}{\mu} u' \left(\frac{\beta}{\mu} \frac{m}{\rho} \right) = 1, \tag{16}$$

and (11) holds. Now, in order that interest-bearing assets be plentiful, we require $a \geq (1-\rho)x^*$, i.e. banks must hold sufficient interest-bearing assets in their portfolios to finance surplus-maximizing consumption for monitored depositors. From (11), this gives

$$m \left(\frac{1}{\delta} - 1 \right) \geq (1-\rho)x^*. \tag{17}$$

Now, our assumption that $-x\frac{u''(x)}{u'(x)} < 1$ implies that the demand for currency, m, implied by (16) is increasing in the gross rate of return on money $\frac{1}{\mu}$, which implies that we can write the solution to (16) as $m = m(\mu)$, where $m(\mu)$ is a continuous and strictly decreasing function on the domain $[\beta, \infty)$, with $m(\beta) = \rho x^*$. Note that (17) is not satisfied for $\delta < 0$ (the consolidated government is a net creditor) or for $\delta \geq 1$. From (17), this equilibrium exists in the set

$$\left\{ (\delta, \mu) : \mu > \beta, \ 0 < \delta \leq \frac{m(\mu)}{(1-\rho)x^* + m(\mu)} \right\} \tag{18}$$

Now, in this equilibrium, δ is irrelevant for real quantities of interest. A change in δ, interpreted as a one-time open market operation, is neutral, having no effect on the real interest rate (which is invariant at the rate of time preference) or on consumption in the DM. From (16), the real stock of currency m is invariant to changes in δ, so the price level increases in proportion to any money injected through an open market operation. Note, from (18), that higher inflation, i.e higher μ, implies that the threshold for δ at which interest-bearing assets are not plentiful gets smaller. In general, the consolidated government must issue a sufficiently small quantity of non-interest-bearing currency relative to the total consolidated government debt, in order for interest-bearing assets to be sufficiently plentiful in equilibrium.

3.3.3 Equilibrium With Scarce Interest-Bearing Assets

In this equilibrium, $\frac{1}{\mu} < r < \frac{1}{\beta}$, so the nominal interest rate is positive, as in the equilibrium with plentiful interest-bearing assets. The difference here is that interest-bearing assets are also scarce, and this implies that exchange is inefficient in the DM in both non-monitored and monitored meetings. An equilibrium of this type consists of (m, a, r) solving the market-clearing condition (11), and the two first-order conditions (6) and (7), or (16) and

$$\beta r u' \left(\frac{\beta r a}{1-\rho} \right) = 1. \tag{19}$$

Here, equation (16) solves for the real quantity of currency, $m(\mu)$, as in the previous subsection. Then, equations (11) and (19) yield

$$\beta r u' \left[\frac{\beta r m(\mu) \left(\frac{1}{\delta} - 1 \right)}{1-\rho} \right] = 1. \tag{20}$$

which solves for r given (δ, μ). Nonnegative consumption in the DM implies, from (20), that $0 < \delta < 1$ is necessary for existence of the equilibrium. Given this, necessary and sufficient conditions for the existence of this equilibrium are that the solution to (20) satisfy $\frac{1}{\mu} < r < \frac{1}{\beta}$. Then, since $-x\frac{u''(x)}{u'(x)} < 1$ implies the left-hand side of equation (20) is increasing in r, this equilibrium therefore exists in the set

$$\left\{ (\delta, \mu) : \mu > \beta, \ \frac{m(\mu)}{(1-\rho)x^* + m(\mu)} < \delta < \rho \right\} \tag{21}$$

Now, from (20), since $-x\frac{u''(x)}{u'(x)} < 1$, it is straightforward to show that r is strictly decreasing in both δ and μ. First, an increase in δ is essentially a one-time open market purchase that increases the proportion of currency relative to interest bearing nominal government debt. This leaves the real quantity of currency m unaffected, i.e. the price level rises in proportion to the increase in the nominal currency stock. However, money is not neutral, as the real interest rate r decreases permanently. This occurs due to an *illiquidity effect*. The quantity of government bonds has decreased in real terms, and therefore there is a smaller quantity of liquid assets available to back the bank liabilities that are being traded in monitored exchanges in the DM. As government bonds have become more scarce, they demand a larger liquidity premium, and the real interest rate falls.

Second, if μ increases then, as $m'(\mu) < 0$, this reduces the price level, thus reducing the total real stock of consolidated government debt outstanding at each date. As in the case of an open market purchase, this makes government bonds more scarce in their role as backing for the bank liabilities supporting exchange.

These results are illustrated in Figure 1. Given a monetary policy (δ_1, μ_1) the demand curve for interest-bearing assets a is D_1 determined by (19), subject to the upper and lower bounds,

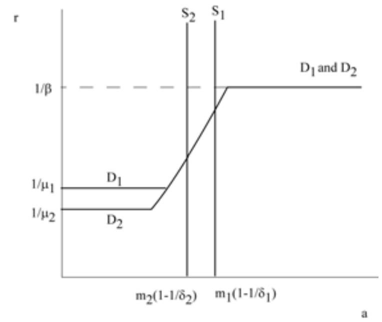

Figure 1: The Illiquidity Effect

$\frac{1}{\mu_1}$ and $\frac{1}{\beta}$, respectively, on the real gross interest rate. We depict a case where the supply curve S_1 yields an equilibrium with scarce interest-bearing assets with $\frac{1}{\mu_1} < r < \frac{1}{\beta}$. An alternative policy (δ_2, μ_2) with $\delta_2 \geq \delta_1$ and $\mu_2 \geq \mu_1$ implies a shift to the left in the supply curve to S_2 and a shift in the demand curve to D_2, producing decreases in a and r.

3.3.4 Friedman Rule Equilibrium

A Friedman rule equilibrium, with $\frac{1}{\mu} = r = \frac{1}{\beta}$, obtains for any δ, whenever $\mu = \beta$. Thus, there are many ways to implement the Friedman rule. A typical approach in much of the literature on monetary economics (e.g. Lagos-Wright 2005) is to assume $\delta = 1$, in which case the consolidated government effectively issues only currency, and at the Friedman rule lump sum taxes cause the currency stock to contract over time. Another Friedman rule policy in the set of alternatives is to have $\delta \to \infty$, which could be achieved if the fiscal authority issues no debt and the central bank issues the entire stock of money through central bank lending. In this case, the central bank lends out the entire stock of currency at $t = 0$ and then retires currency over time using the net interest on the central bank loans, while keeping the real stock of central bank loans constant over time.

3.3.5 Existence

Figure 2 shows the types of equilibria that exist for given policies (δ, μ). From our analysis above, an equilibrium does not exist if $\delta < 0$ and $\mu > \beta$, since this must imply that the total consolidated government debt is less than or equal to zero, which cannot support exchange in the DM with nonnegative consumption. However, an equilibrium exists for any δ when $\mu = \beta$, and for any $\mu > \beta$ when $\delta > 0$. In Figure 2 note that, if a stationary equilibrium exists, it is unique. Further, given μ, as we increase δ we move from the equilibrium with plentiful interest bearing

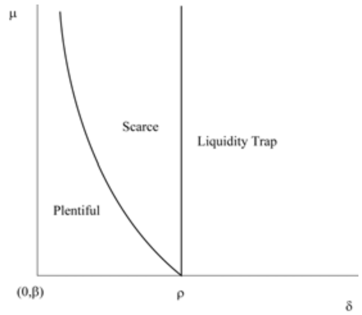

Figure 2: Equilibria with Money and Government Bonds

assets to the equilibrium with scarce interest bearing assets to the liquidity trap equilibrium. For any μ, we obtain the liquidity trap equilibrium as long as δ is sufficiently large.

3.3.6 Optimality

Now, what is an optimal allocation in this economy? As in most Lagos-Wright (2005) setups, if we weight utilities of buyers and sellers equally in an aggregate welfare measure, then CM activity drops out, and welfare is proportional to the total surplus in DM matches, which is given by

$$W = \rho[u(x_n) - x_n] + (1-\rho)[u(x_m) - x_m], \tag{22}$$

where x_n is consumption in non-monitored meetings, and x_m is consumption in monitored meetings. Any Friedman rule allocation is optimal, since $x_n = x_m = x^*$ in a Friedman rule equilibrium, which maximizes surplus W. But, in any equilibrium with $\mu > \beta$, we have $x_n < x^*$, and in equilibria with scarce interest-bearing assets or in a liquidity trap equilibrium, we also have $x_m < x^*$.

Now, in addition to yielding an optimal allocation of resources, the Friedman rule, however implemented, implies that there exists an equilibrium where banks do not exist. In this equilibrium all goods in the DM are purchased with currency, with government bonds held from one CM until the next CM, and not traded in the DM. What is wrong with this picture? Clearly, it does not make much sense as a description of what an optimal financial arrangement might look like in the real world. It cannot be optimal to be making all transactions using currency, and the reasons should be obvious. While currency has some very useful attributes - settlement of debts is immediate, trade under anonymity is possible, no sophisticated information technology is necessary - exchange using currency is subject to some potentially severe inefficiencies. First, currency can be stolen. We all know why it is not a good idea to carry large sums of cash in our pockets, or to send it in the mail. Second, it is costly for the government to maintain the stock of currency. Worn-out notes must be shredded and replaced, armored trucks are needed to transport old currency to the local Federal Reserve Bank and to distribute the new currency to financial institutions, and the currency must be designed to thwart counterfeiters in an efficient way. Third, there are social losses from the time and effort expended by counterfeiters and thieves. Fourth, the existence of currency makes illegal activities, including tax evasion and trade in illegal commodities, less costly.

The Friedman rule has always been somewhat of a puzzle in monetary economics. Most basic monetary models imply that the Friedman rule is optimal, but we never observe central banks adopting monetary policy rules that imply zero nominal interest rates forever. Our contention is that most monetary models leave out some critical elements, which are the costs associated with currency exchange, as we outlined in the previous paragraph. We will explore this further in what follows.

3.3.7 Interest on Reserves

Up to now, we have considered a setup where it is necessary for the nominal interest rate to be zero for reserves to be held by banks. However, given that we want our model to apply generally, we need to consider what happens if outside money, held in reserve accounts with the central bank, bears interest. Some central banks in the world, including the European Central Bank and the Bank of Canada, have paid interest on reserves for some time, and the U.S. Congress recently approved the payment of interest on reserve accounts held at Federal Reserve Banks. The Fed has been paying interest on reserves since October 2008 at 0.25%.

Let E_t denote the quantity of reserves, where reserves are perfect substitutes for government bonds in equilibrium, bearing a real gross rate of return of r. The model does not contain a financial transactions role for reserves, so these interest bearing accounts with the monetary authority are no different from government bonds, which are interest-bearing accounts with the fiscal authority. This does not do much violence to current reality in the United States as, under the present conditions (Summer 2011) most of the stock of reserves held by financial institutions in the United States is not being used in inter-bank transactions, and the marginal transactions value of reserves is essentially zero. With interest-bearing reserves, the monetary authority determines the total quantity of outside money, currency plus reserves, or $M_t + E_t$, and banks and buyers jointly determine how the outside money is split between currency and reserves.

To make this more explicit, let η denote the ratio of outside money to outstanding nominal liabilities of the consolidated government, i.e.

$$M_t + E_t = \eta(M_t + B_t + E_t) \tag{23}$$

for all t. Now, define δ just as before, as the ratio of currency to the total outstanding nominal debt of the consolidated government, i.e.

$$M_t = \delta(M_t + B_t + E_t). \tag{24}$$

Now, in an equilibrium with $E_t > 0$, i.e. a strictly positive quantity of nominal interest-bearing reserves, the monetary authority chooses (r, η, μ), where r is the gross real interest rate on reserves. Equivalently, we could have the monetary authority choose (R, η, μ), where R is the gross nominal interest rate on reserves and $r = \mu R$ (the Fisher relation). In equilibrium, government bonds and reserves must bear the same rate of interest, and δ is now an endogenous variable.

Then, if the interest rate on reserves is set so that $\frac{1}{\mu} = r < \frac{1}{\beta}$, any $\delta \geq \rho$ is an equilibrium, and from (23) and (24) we require that $\delta \leq \eta$. Therefore, the liquidity trap equilibrium exists if and only if $\eta > \rho$, and equilibrium δ satisfies $\rho \leq \delta < \eta$, but is otherwise indeterminate, though the consumption allocation is the same in each equilibrium in the continuum.

If the interest rate on reserves is set so that $\frac{1}{\mu} < r = \frac{1}{\beta}$ then, if an equilibrium exists it will be one with plentiful interest-bearing assets. Then, from (18), δ must satisfy

$$0 < \delta \leq \frac{m(\mu)}{(1-\rho)x^* + m(\mu)},$$

and, as in the liquidity trap equilibrium, $\delta < \eta$, so an equilibrium of this type can exist if and only if $\eta > 0$, and if that condition holds then δ is indeterminate, with

$$\delta \in \left(0, \min\left[\eta, \frac{m(\mu)}{(1-\rho)x^* + m(\mu)}\right]\right].$$

Finally, the most interesting case is the one where the interest rate on reserves is set to satisfy $\frac{1}{\mu} < r < \frac{1}{\beta}$. Then (20) solves for δ given r and μ. From (20), η must satisfy

$$\beta r u'\left[\frac{\beta r m(\mu)\left(\frac{1}{\eta} - 1\right)}{1-\rho}\right] > 1. \tag{25}$$

in order to have $\delta < \eta$ in equilibrium. Now, in this equilibrium a one-time open market purchase, i.e. a change in η, has no effect at the margin, since this is just a swap of interest bearing reserves for interest-bearing government bonds. However, changing r, the gross real interest rate on reserves, matters. If r decreases with η held fixed, then this will result in an increase in δ, from (20). Thus, a reduction in the interest rate on reserves reduces all market interest rates, and serves to cause an increase in the ratio of currency to total nominal consolidated-government liabilities. In conjunction with a market interest rate that reflects an increase in the scarcity of interest-bearing assets, the quantity of reserves shrinks, in real and nominal terms. Intuitively, there is essentially a decrease in the demand for reserves as the interest rate on reserves falls.

There is an important lesson here that relates to the current predicament of the Fed, which has issued large quantities of reserve balances, well in excess of reserve requirements. To traditional monetarists, it might appear puzzling that inflation is low in the United States and Europe in spite of large increases in outside money. However, our model tells us that large increases in outside money accomplished by increasing interest-bearing reserves can have no effect on prices (increasing η is irrelevant in general) so this is not puzzling. Further, our model tells us that the existence of a large quantity of excess reserves is not a problem for controlling the price level. For example, suppose that there exists an equilibrium with $\frac{1}{\mu} < r < \frac{1}{\beta}$ given (r_1, η, μ), where η satisfies (25) and (20) solves for $\delta = \delta_1$ given $r = r_1$ and μ. Holding η and μ constant, if the monetary authority increases the interest rate on reserves above r_1, then from (20) δ must decrease below δ_1, which implies a decrease in the price level. The central bank has all the control it needs by using the interest rate on reserves as a policy instrument, in spite of the fact that open market operations have no effect with a positive quantity of excess reserves in the system.

4 Non-Passive Fiscal Policy

Now that we have a basic working knowledge this model, we can extend it to make it more interesting and applicable to current problems. There are two respects in which our model can be modified to extract some additional insight. First, given the relationship in typical developed economies between the central bank and the fiscal authority, there are better ways to think about monetary policy than to have fiscal policy be purely passive. Second, as was discussed above, the model's predictions for optimal policy, like those of most mainstream monetary models, are problematic. The Friedman rule is optimal in this model, but running the Friedman rule supports equilibria where currency is the dominant means of payment, which does not seem too helpful. In this section, we start by adding non-passive fiscal policy. Then, in the next section, we add costs of currency exchange to the model. We then show how these two elements - non-passive fiscal policy and costs of currency exchange - working together, make the monetary policy problem and its solution interesting.

In a non-passive fiscal policy regime in this model, what is critical for the central bank is the deficit that the consolidated government needs to finance each period. Assume that the fiscal authority fixes the deficit at a constant level σ forever, in real terms. To do this, assume that the fiscal authority also commits to levying lump-sum taxes in each period to pay the net interest on the outstanding government debt. As before, the central bank sets δ, the ratio of currency to total outstanding government debt according to (8). Now, however, instead of (9), lump sum taxes to pay the interest on the government's debt are determined passively, i.e.

$$\tau_t = \frac{\phi_t M_{t-1}(1-\delta)}{\delta}(\mu r_{t+1} - 1), \tag{26}$$

for $t = 1, 2, 3, ...$, and we also assume that τ_0 is determined as in equation (10). Also, most importantly, from (9) the deficit is financed each period by the issue of interest-bearing debt and currency, according to

$$\sigma = \frac{\phi_t M_t}{\delta}\left(1 - \frac{1}{\mu}\right), \tag{27}$$

for $t = 1, 2, 3, ...$. This implies that δ and μ cannot be set independently. An interpretation is that the fiscal authority fixes the deficit and issues whatever nominal debt is necessary to finance it given the setting of the monetary policy instrument δ by the central bank. An important assumption here is that the central bank can choose $\delta > 0$ and the fiscal authority will follow the central bank's lead and issue nominal debt each period, while if $\delta < 0$ then the fiscal authority lends to the private sector.

Now, in a stationary equilibrium, $m = \phi_t M_t$ (currency demand equals currency supply in the CM), so (27) gives

$$\sigma = \frac{m}{\delta}\left(1 - \frac{1}{\mu}\right). \tag{28}$$

Now, for given (δ, μ), the equilibrium is determined in exactly the same way as in the previous section, but now (δ, μ) is constrained by (28), where m is an endogenous variable which in turn depends, in general, on (δ, μ).

First, in a liquidity trap equilibrium, where $\frac{1}{\mu} = r < \frac{1}{\beta}$, substituting for m in equation (14) and rearranging, we get

$$u'\left(\frac{\beta\sigma}{\mu - 1}\right) = \frac{\mu}{\beta}, \tag{29}$$

which determines μ. Let $g(\mu)$ denote the left-hand side of equation (29). Then, $g(1) = 0$, $g(\infty) = \infty$, and it is straightforward to show that, since $-x\frac{u''(x)}{u'(x)} < 1$, $g(\cdot)$ is strictly concave with $g'(0) = \infty$ and $g'(\infty) = 0$. Thus, if a solution exists to (29) then, generically, there are two solutions. Further, if $\sigma \leq \sigma^*$ for some critical $\sigma^* > 0$, then there is a solution to (29), but otherwise there is not. Recall that in a liquidity trap equilibrium, monetary policy is irrelevant, i.e. changing δ (at the margin) has no effect on quantities and prices. In a liquidity trap, μ is the gross rate at which total government liabilities have to grow to finance the deficit, and monetary policy does not matter, as open market operations are just swaps of identical assets.

Second, in an equilibrium with plentiful interest-bearing assets, where $\frac{1}{\mu} < r = \frac{1}{\beta}$, or in an equilibrium with scarce interest-bearing assets, where $\frac{1}{\mu} < r < \frac{1}{\beta}$, substituting for m from (28) in (16) we get

$$u'\left(\frac{\beta\sigma\delta}{(\mu-1)\rho}\right) = \frac{\mu}{\beta}, \tag{30}$$

and equation (30) determines the set of policies (δ, μ) that permit the fiscal authority to just finance its deficit. Here, for given δ and σ, the function on the left-hand side of (30) has the same properties as for equation (29). Thus, given δ, if a solution for μ exists that solves (30), there will be two solutions, generically. For a solution to exist which actually represents either an equilibrium with plentiful interest-bearing assets or scarce interest-bearing assets, requires that $\delta \in (0, \min(\rho, \sigma^*\rho)]$

Finally, in a Friedman rule equilibrium, $\mu = \beta$ and, given any δ, there exists an equilibrium where agents hold a sufficient quantity of consolidated-government liabilities that the deficit is financed each period, for any $\sigma > 0$. Just as in the passive-fiscal-policy case, a Friedman rule yields an efficient equilibrium allocation.

5 Costs Associated With Currency

Now, recall that a key point, made above, is that it is important to take into account the costs of operating a currency system. To illustrate the idea, we will take the simplest possible route, which permits us to use the analysis up to this point to full advantage. Suppose that a fraction v of non-monitored exchanges in the DM, i.e. a fraction ρv of total meetings in the DM, are deemed by society to be of no social value. This is a simple approach to capturing the fact that a significant fraction of the stock of currency is being used as a medium of exchange in illegal trades. For the United States, a large fraction of the value of Federal Reserve Notes is held outside US borders, so this approach can also represent, in a rough way, the idea that we may not care about the welfare of foreign holders of US currency. To model the latter formally, however, would require a full-blown model of international exchange, which we do not want to get into here.

As a result, our welfare measure, rather than (22), is

$$W = \rho(1-\upsilon)u(x_n) - \rho x_n + (1-\rho)[u(x_m) - x_m], \tag{31}$$

where, relative to (22), we have subtracted the utility from illegal consumption, $\rho\upsilon u(x_n)$.

Now, the beauty of this approach to modeling the costs of a currency system is that quantities traded in the DM, prices, and interest rates, are all invariant to υ. This parameter then only matters for welfare and for the determination of an optimal monetary policy.

5.1 Optimal Policy

First suppose that, given the welfare measure (31), we were free to choose x_n and x_m to maximize welfare. Then, we would choose $x_m = x^*$ (surplus-maximizing exchange in monitored DM meetings) and $x_n = \hat{x}$, where $\hat{x}$ solves

$$u'(\hat{x}) = \frac{1}{1-\upsilon},$$

which implies $\hat{x} < x^*$. Now, if fiscal policy is purely passive, then from (18) the policymaker can choose any policy

$$\left\{(\delta,\mu) : \mu = \frac{\beta}{1-\upsilon},\ 0 < \delta \leq \frac{m\left(\frac{\beta}{1-\upsilon}\right)}{(1-\rho)x^* + m\left(\frac{\beta}{1-\upsilon}\right)}\right\} \tag{32}$$

and this is the policy that maximizes welfare, as it supports an equilibrium with $x_n = \hat{x}$ and $x_m = x^*$. Thus, with passive fiscal policy, the ratio of currency to total nominal government liabilities must be small enough that there is a sufficiently large quantity of interest-bearing government debt to support surplus-maximizing exchange in monitored meetings in the DM. As well, the rate of growth in total nominal government debt must be sufficiently high that the resulting inflation optimally taxes socially undesirable exchange using currency in the DM. Note that the optimal inflation rate rises with υ, where υ is a measure of the extent of socially undesirable transactions.

Now, what is much more interesting is the case where fiscal policy is non-passive. To keep things simple, restrict attention to the case where $\sigma > 0$, so that the fiscal authority runs a perpetual deficit. First, consider the case where $0 < \upsilon \leq 1-\beta$. Then, from (30), a monetary policy that achieves $\mu = \frac{\beta}{1-\upsilon}$ is not feasible since these requires $\delta < 0$, and an equilibrium does not exist when $\delta < 0$. Then, an optimal monetary policy is either a Friedman rule, with $\mu = \beta$, or $\mu \to 1$ from above, depending respectively on whether $0 < \upsilon \le \hat{\upsilon}$ or $\hat{\upsilon} \le \upsilon \le 1 - \beta$, where

$$\hat{\upsilon} = 1 - \left[\frac{x^* - \tilde{x}}{u(x^*) - u(\tilde{x})}\right]$$

with $u'(\tilde{x}) = \frac{1}{\beta}$ determining $\tilde{x}$.

Next, there are cases where, even with non-passive fiscal policy, the optimal monetary policy with passive fiscal policy is feasible. A necessary condition for feasibility in this sense is $\upsilon > 1 - \beta$. Then, using (18) and (30), an optimal monetary policy given by (32) can be supported in equilibrium if and only if $\upsilon \in (1 - \beta, \upsilon^*]$, where υ^* is the unique solution to

$$u'\left[\frac{\rho\beta(1 - \upsilon^*)\sigma + (1 - \rho)(\beta - 1 + \upsilon^*)x^*}{\rho(\beta - 1 + \upsilon^*)}\right] = \frac{1}{1 - \upsilon^*},$$

and note that $\upsilon^* < \frac{\beta\sigma + (1-\beta)(1-\rho)x^*}{\beta\sigma + (1-\rho)x^*} < 1$.

Finally, if $\upsilon > \upsilon^*$, then an optimal monetary policy given by (32) is not feasible. In this case, consider the equilibrium with scarce interest-bearing assets where the policy (δ, μ), is constrained by (30). Consumption quantities in the DM are, from (16), (20), and (30),

$$x_n = \frac{\beta\sigma\delta}{(\mu - 1)\rho}, \tag{33}$$

$$x_m = \frac{\beta r\mu\sigma(1 - \delta)}{(1 - \rho)(\mu - 1)}, \tag{34}$$

where r is determined by

$$\beta r u'\left[\frac{\beta r\mu\sigma(1 - \delta)}{(1 - \rho)(\mu - 1)}\right] = 1. \tag{35}$$

Now, think of the central bank's policy problem as choosing μ, which then determines δ from (30). Then, suppose that we calculate the derivative of welfare, W, with respect to μ, evaluated for the policy setting that yields an equilibrium with scarce interest-bearing assets. Also the derivative is evaluated where $r = \frac{1}{\beta}$, so that smaller μ would imply that the policy would yield an equilibrium with plentiful interest-bearing assets. Then, for this policy, from (31), (33), and (34), we get

$$\frac{\partial W}{\partial \mu} = \frac{\rho}{\beta u''\left(\frac{\beta\sigma\delta}{(\mu-1)\rho}\right)}\left[(1 - \upsilon)\frac{\mu}{\beta} - 1\right] > 0,$$

since $\upsilon > \upsilon^*$ implies that $\mu < \frac{\beta}{1-\upsilon}$ in the expression above. Therefore, in this case an optimal monetary policy supports either an equilibrium with scarce interest-bearing assets or one with a liquidity trap. If the optimal monetary monetary policy implies an equilibrium with scarce interest-bearing assets, then

$$\frac{\partial W}{\partial \mu} = \Omega_n + \Omega_m = 0 \tag{36}$$

at the optimum, where Ω_n (Ω_m) denotes the effect on the surplus in non-monitored (monitored) exchange in the DM of a change in policy. Here,

$$\Omega_m = \frac{\rho}{\beta u''\left(\frac{\beta\sigma\delta}{(\mu-1)\rho}\right)}\left[(1-\upsilon)\frac{\mu}{\beta} - 1\right], \tag{37}$$

$$\Omega_n = -\frac{(1-\rho)}{\beta r^2 u''\left(\frac{\beta^2 r\sigma(1-\delta)}{(\mu-1)(1-\rho)}\right)}\frac{\partial r}{\partial \mu}. \tag{38}$$

Therefore, if $\frac{\partial r}{\partial \mu} < 0$, then $\mu < \frac{\beta}{1-\upsilon}$ at the optimum. In this case, a higher money growth rate (and thus a higher inflation rate) is beneficial at the margin, as this taxes currency transactions that have no social value. However, higher inflation also reduces the real interest rate, which reflects an increasing scarcity of liquid assets in monitored exchange in the DM. The more this asset scarcity effect matters, the lower will be the optimal inflation rate. If δ increases with μ, which is possible from (30), then this is a necessary condition for $\frac{\partial r}{\partial \mu} > 0$. Then, if $\frac{\partial r}{\partial \mu} > 0$ at the optimum, we will have $\mu > \frac{\beta}{1-\upsilon}$, since higher inflation makes liquid assets less scarce in monitored exchange in the DM.

Now, it is also possible that, with $\upsilon > \upsilon^*$, an optimal monetary policy supports a liquidity trap equilibrium. A necessary condition for this, from (36)-(38), is

$$\rho(1-\upsilon)\frac{\mu}{\beta} - \rho - (1-\rho)\mu^2\frac{\partial r}{\partial \mu} < 0,$$

which will hold for υ and ρ sufficiently large.

The key point here is that, so long as the social costs of currency exchange are significant, i.e. υ is sufficiently large, then a Friedman rule is not optimal. This result does not require a non-passive fiscal policy, as with passive fiscal policy the optimal gross inflation rate is $\mu = \frac{\beta}{1-\upsilon}$, which deviates from the Friedman rule by an amount which is increasing in υ. With non-passive fiscal policy, the monetary authority will in general be faced with a tradeoff. Higher money growth and inflation will tax socially undesirable exchange which uses currency. However, financing the government's deficit with a higher inflation rate may also imply a larger ratio of currency to total nominal government debt, which in turn reduces the quantity of liquid assets supporting monitored exchange in the DM.

6 Private Liquidity

A useful modeling extension, and one that permits us to address issues related to the recent financial crisis and the monetary policy response to the crisis, is to include the private production of liquid assets. There are alternative approaches to doing this, for example we could follow what is done in Lagos and Rocheteau (2008), and include trade in claims to capital in the model. Here, we will permit the private production of liquidity by way of a costly-state-verification delegated-monitoring intermediary structure similar to that in Williamson (1987). This is a rich framework that permits us to incorporate credit allocation, debt contracts, and default premia in the model, all of which are important for studying the financial crisis.

6.1 Intermediated Credit: Delegated Monitoring

The extension of the model will be quite straightforward. We need to add some economic agents, and additional details, but we will be able to use to full advantage the analysis we have done thus far.

We will assume that, during the CM of each period, a continuum of entrepreneurs with mass α is born, and each lives until the CM of the following period. An entrepreneur born in the CM of period t consumes only in the CM of period $t+1$, is risk neutral, and receives no endowment during his or her lifetime. Each entrepreneur has access to an investment project. This project is indivisible, requires one unit of the consumption good in the CM of period t to operate, and yields a return of w in the CM of period $t+1$, where w is distributed according to the distribution function $F(w)$, with associated density function $f(w)$, which is strictly positive only on $[0, \bar{w}]$, where $\bar{w} > 0$. Assume also that $f(\cdot)$ is continuously differentiable. Investment project returns are independent across entrepreneurs. The return w is private information to the entrepreneur, but subject to costly state verification, i.e. any other individual can bear a fixed cost and observe w ex post. The verification cost γ is entrepreneur-specific, and $G(\gamma)$ denotes the distribution of verification costs across entrepreneurs, with $\gamma \geq 0$. One way to interpret the environment is that an entrepreneur, when born, receives a draw from the distribution $G(\cdot)$, which is his or her verification cost. Then, if the entrpeneur's project is funded, the return w is a draw from the distribution $F(\cdot)$, and w is independent of γ.

We will assume that stochastic verification is not feasible, and that entrepreneurs are economic agents who are subject to full commitment. Then, as in Williamson (1987), an efficient lending arrangement is for individual entrepreneurs to act as perfectly-diversified financial intermediaries. Efficient and incentive-compatible loan contracts with entrepreneurs take the form of non-contingent debt. That is, the financial intermediary observes the verification cost γ associated with the entrepreneur in the CM and offers him or her a contract that specifies a non-contingent payment R that the entrepreneur must make to the intermediary in the next CM. If the entrepreneur cannot make the loan payment, then default occurs, the intermediary incurs the verification cost γ, observes the return w, and seizes the entrepreneur's output. As shown in Williamson (1987), the expected payoff to the intermediary from the loan contract, as a function of the non-contingent payment R and the verification cost γ, is then given by

$$\pi(R,\gamma) = R - \gamma F(R) - \int_0^R F(w)dw \tag{39}$$

Then, letting $R(\gamma)$ denote the gross real loan interest rate on a loan to an entrepreneur of type γ, equation (39) allows us to define the default premium faced by an entrepreneur of type γ, which is

$$D(\gamma) = \gamma F[R(\gamma)] + \int_0^{R(\gamma)} F(w)dw. \tag{40}$$

As above, we focus on stationary equilibria where all real variables are constant for all t, including the gross real interest rate on government debt, r. A bank in our model now acquires deposits from buyers in the CM, and acquires a portfolio of currency, nominal government bonds, and loans to entrepreneurs. In equilibrium the bank, which is perfectly diversified (this requires only that it hold a positive mass of loans to entrepreneurs), receives a certain one-period return r per unit lent to entrepreneurs. Further, in equilibrium the expected payoff will be the same for each loan made by the bank, so

$$r = R(\gamma) - \gamma F[R(\gamma)] - \int_0^{R(\gamma)} F(w)dw \tag{41}$$

for each entrepreneur who receives a loan. Assume that $-\gamma f'(w) - f(w) < 0$ for all $w \in [0,\bar{w}]$ and for all $\gamma \geq 0$, which implies that $\pi(R,\gamma)$ is strictly concave in R for $R \in [0,\bar{w}]$ and attains a maximum for $R = \hat{R}(\gamma) < \bar{w}$, where

$$1 - \gamma f\left[\hat{R}(\gamma)\right] - F\left[\hat{R}(\gamma)\right] = 0. \tag{42}$$

In equilibrium, there is a marginal entrepreneur with verification cost γ^* and facing the gross loan interest rate R^*, where, from (42),

$$1 - \gamma^* f(R^*) - F(R^*) = 0, \tag{43}$$

so that the gross loan interest rate faced by the marginal entrepreneur maximizes the expected return to the financial intermediary given the marginal entrepreneur's verification cost γ^*. Further, the bank earns an expected return r from lending to the marginal borrower, or from (39),

$$r = R^* - \gamma^* F\left(R^*\right) - \int_0^{R^*} F(w)dw \tag{44}$$

Then, each entrepreneur who receives a loan has $\gamma \leq \gamma^*$, and if $\gamma < \gamma^*$ then $R(\gamma) < \hat{R}(\gamma)$. Entrepreneurs with $\gamma > \gamma^*$ do not receive loans as, even if $R(\gamma) = \hat{R}(\gamma)$ for one of these agents, lending would involve an expected loss for the bank.

Then, the total quantity of loans extended by banks during the CM is given by

$$L = \alpha G(\gamma^*). \tag{45}$$

Therefore, given the certain return on lending r, (43), (44), and (45) determine the loan quantity L, the verification cost of the marginal borrower, γ^*, and the gross loan interest rate faced by the marginal borrower R^*. It is straightforward to show that the quantity of lending L decreases when the payoff per unit lent by the bank, r, increases. This is because, from (41), the loan interest rate for each entrepreneur receiving a loan will increase, and it will be unprofitable for a bank to lend to a formerly marginal entrepreneur. Further, since the loan interest rate will be higher for each creditworthy entrepreneur when the real interest rate increases, from (40) each of these creditworthy entrepreneurs will be faced with a higher default premium.

Given (43)-(45), we can write $L = L(r)$, where $L(\cdot)$ is a decreasing function. For some of our results, all we will need is the reduced form $L(r)$. However, the rich detail in how entrepreneurs' investment projects are funded and the structure of loan interest rates and default premia across projects, will be particularly useful for understanding issues related to the financial crisis and central bank credit market interventions.

The costly-state-verification structure gives us production of private liquid assets which, though they are information-intensive and bear idiosyncratic risk, are essentially perfect substitutes for safe government debt from the point of view of financial intermediaries. This setup also gives us a set of determinants of the supply of private liquid assets. The quantity of lending depends on how many potential investment projects exist (α), how costly it is to collect on debts (the distribution $G(\gamma)$), how productive the investment projects are ($F(w)$), and on the opportunity cost of lending (r).

6.2 Equilibria with Private Liquidity

In defining an equilibrium given what we have added to the model, nothing changes other than the asset market-clearing condition (11) which becomes

$$a = m\left(\frac{1}{\delta} - 1\right) + L(r), \tag{46}$$

i.e. the demand for interest-bearing assets a, by banks, must equal the real quantity of government bonds plus lending to entrepreneurs. As before, an equilibrium could be one of four types: a liquidity trap equilibrium, an equilibrium with plentiful interest-bearing assets, an equilibrium with scarce interest-bearing assets, or a Friedman rule equilibrium. These equilibria are characterized in essentially the same way, but private liquidity alters the regions of the policy parameter space where particular types of equilibria exist. Recall that the policies we examine are determined by (δ, μ), where δ is the ratio of currency to total nominal consolidated government debt and μ is the gross rate of growth in that debt. The following three propositions give a full characterization of existence of equilibrium with private liquidity. First define δ_a by

$$\delta_a \equiv \frac{\rho x^*}{x^* - L\left(\frac{1}{\beta}\right)}.$$

Second, recall that $m = m(\mu)$ is the solution to (16), and let μ_a denote the solution to

$$\frac{m(\mu_a)}{\rho} = L\left(\frac{1}{\mu_a}\right),$$

if $x^* > L\left(\frac{1}{\beta}\right)$. Third, let $\mu_b(\delta)$ denote the solution to

$$\frac{1}{\delta} = \frac{1}{\rho} - \frac{L\left(\frac{1}{\mu_b(\delta)}\right)}{m(\mu_b(\delta))}.$$

Finally, let $\mu_c(\delta)$ denote the solution to

$$\frac{1}{\delta} = 1 + \frac{L\left(\frac{1}{\beta}\right) - (1-\rho)x^*}{m(\mu_c(\delta))}.$$

Proposition 1: If

$$L\left(\frac{1}{\beta}\right) < (1-\rho)x^*,$$

then (i) a liquidity trap equilibrium exists if and only if $\delta \geq \delta_a$ and $\mu \in (\beta, \mu_b(\delta)]$ or $\delta < 0$ and $\mu \in (\mu_a, \mu_b(\delta)]$; (ii) an equilibrium with plentiful interest-bearing assets exists if and only if $0 < \delta \leq \delta_a$ and $\mu \in (\beta, \mu_c(\delta)]$; (iii) an equilibrium with scarce interest-bearing assets exists if and only if $0 < \delta \leq \delta_a$ and $\mu > \mu_c(\delta)$, or $\delta > \delta_a$ and $\mu > \mu_b(\delta)$, or $\delta < 0$ and $\mu > \mu_b(\delta)$.

Proof. See the appendix. ■

Proposition 2: If

$$(1-\rho)x^* < L\left(\frac{1}{\beta}\right) < x^*,$$

then (i) a liquidity trap equilibrium exists if and only if $\delta \geq \delta_a$ and $\mu \in (\beta, \mu_b(\delta)]$ or $\delta < 0$ and $\mu \in (\mu_a, \mu_b(\delta)]$; (ii) an equilibrium with plentiful interest-bearing assets exists if and only if $0 < \delta \leq \delta_a$ and $\mu > \beta$ or $\delta > \delta_a$ and $\mu \in [\mu_c(\delta), \infty)$ or $\delta < 0$ and $\mu \in [\mu_c(\delta), \infty)$; (iii) an equilibrium with scarce interest-bearing assets exists if and only if $\delta > \delta_a$ and $\mu \in (\mu_b(\delta), \mu_c(\delta))$ or $\delta < 0$ and $\mu \in (\mu_b(\delta), \mu_c(\delta))$.

Proof. See the appendix. ■

Proposition 3: If

$$x^* < L\left(\frac{1}{\beta}\right),$$

then (i) a liquidity trap equilibrium exists if and only if $\delta_a \leq \delta < 0$ and $\mu \in (\beta, \mu_b(\delta)]$; (ii) an equilibrium with plentiful interest-bearing assets exists if and only if $\delta > 0$ and $\mu > \beta$ or $\delta_a < \delta < 0$ and $\mu \geq \mu_c(\delta)$ or $\delta \leq \delta_a$ and $\mu > \beta$; (iii) an equilibrium with scarce interest-bearing assets exists if and only if $\delta_a < \delta < 0$ and $\mu \in (\mu_b(\delta), \mu_c(\delta))$.

Proof. See the appendix. ■

The intuition behind Propositions 1 to 3 is that what matters for the existence of particular equilibria is the private economy's ability to produce liquid assets, relative to the demand for those assets in particular transactions. In particular, $L\left(\frac{1}{\beta}\right)$ is a key measure of the private economy's capacity for producing liquid assets and x^* is the efficient quantity traded in a DM transaction. Thus, $L\left(\frac{1}{\beta}\right) - x^*$ is a measure of liquid-asset capacity relative to the demand for liquid assets in all DM exchange, and $L\left(\frac{1}{\beta}\right) - (1-\rho)x^*$ is a measure of liquid-asset capacity relative to the demand for liquid assets in monitored DM exchange only.

Figures 3 through 5 depict what is represented in Propositions 1 through 3, respectively, showing the equilibria that exist given policy settings (δ, μ). Note first that, due to the ability of the private sector to produce liquid assets, the region of the parameter space with plentiful interest-bearing assets expands (compare to Figures 3-5 to Figure 1). Second, equilibria can now exist when the consolidated government is a net creditor, i.e. when $\delta < 0$, which was not the case without private liquid assets. This result obtains since a sufficiently low interest rate can now induce enough lending and a sufficiently large supply of private liquid assets to offset the government's position as a net creditor, so that there are enough liquid assets to support some exchange in monitored trades in the DM.

A feature of the equilibria that is robust with respect to the introduction of private liquid

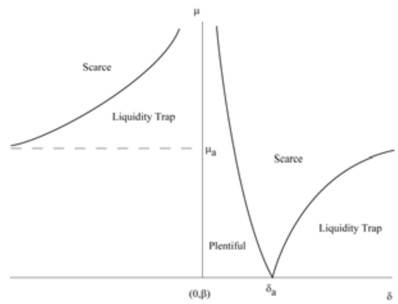

Figure 3: Equilibria with Private Liquidity: $L\left(\frac{1}{\beta}\right) < (1-\rho)x^*$

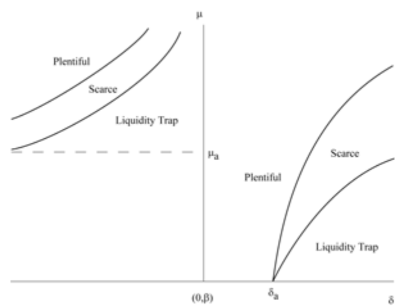

Figure 4: Equilibria with Private Liquidity: $(1-\rho)x^* < L\left(\frac{1}{\beta}\right) < x^*$

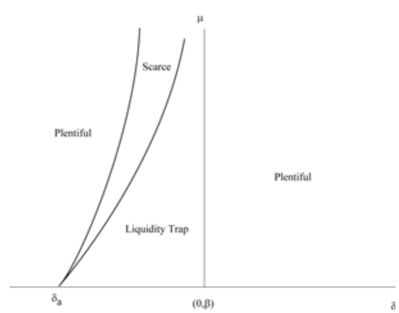

Figure 5: Equilibria with Private Liquidity: $x^* < L\left(\frac{1}{\beta}\right)$

assets is that, for any μ, we can always choose δ so that a liquidity trap equilibrium exists. As well, the qualitative responses of endogenous variables to changes in policy are the same. For example, a change in δ is irrelevant in the liquidity trap equilibrium, an increase in δ reduces the real interest rate in the scarce interest-bearing assets equilibrium, and an increase in δ has no effect on the real interest rate but increases the price level in the plentiful interest-bearing assets equilibrium.

An interesting feature we get by adding private liquid-asset production is that lending will in general vary with policy. For example, in the scarce interest-bearing assets equilibrium, a one-time open market purchase (increase in δ) acts to reduce the real interest rate and increase the quantity of lending to entrepreneurs. Thus, an open market purchase, which reduces the quantity of publicly-provided liquidity (government bonds) that can be exchanged in monitored meetings in the DM, makes liquid assets more scarce, and induces the production of more private liquid assets, which in turn mitigates the scarcity. This is a kind of credit market channel for monetary policy, though it works in a much different way than a conventional credit channel story, in which a liquidity effect of monetary policy reduces the real interest rate and increases the demand for loans. Here, lending increases because of the *illiquidity effect* discussed earlier. An open market purchase makes a class of liquid assets more scarce, and this makes lending more profitable.

6.3 Financial Crisis Issues

We now have a model that contains some of the key elements that appear to have been important in the financial crisis, so we can proceed to use this framework to say something about recent events and monetary policy issues. We will first show how the model captures some of the qualitative features of the financial crisis in the United States, and then analyze the role of monetary policy in this context. We first look at conventional open market operations, and then address the effects of unconventional asset purchases by the central bank.

6.3.1 Factors Affecting Credit Market Activity, Interest Rates, and Interest Rate Spreads

The financial crisis, beginning in late 2008, was marked by decreases in safe market rates of interest, increases in the interest rate spreads between risky and safe debt, and reductions in credit market activity. How might we capture these effects in our framework with private liquidity? There are several factors that could reduce the private economy's capacity to produce

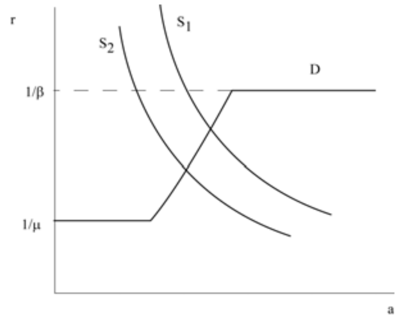

Figure 6: The Illiquidity Effect with Private Assets

liquid assets in this model, and all of these factors act to shift the function $L(r)$. These factors are: (i) changes in the distribution of verification costs across entrepreneurs; (ii) first-order stochastic dominance changes in the distribution of project returns; (iii) changes in the riskiness of project returns. Each of these factors can be interpreted in ways that shed light on the financial mechanism at work in the financial crisis and the 2007-09 recession.

First, a change in the distribution of verification costs, for example a positive first-order-stochastic dominance shift in the distribution function $G(\cdot)$ would have the effect of reducing the quantity of lending for each gross real interest rate r, from (45). In a scarce-interest-bearing-assets equilibrium, this will have an effect as in Figure 6. The supply curve (now interest-elastic due to private liquidity production, as opposed to Figure 1) shifts left from S_1 to S_2, the real interest rate falls in equilibrium, and the quantity of lending decreases. Further, the average default premium, which from (40) is the margin between the gross real interest rate on safe government debt and the average gross loan interest rate, will tend to increase because of the increase in average verification costs.

What interpretation could we give to such a shift in $G(\cdot)$? One interpretation of the verification cost γ is that this is literally bankruptcy costs - the cost of going to court, sorting out what the remaining assets of the entrepreneur are worth, and transferring those assets from the entrepreneur to the bank. A second interpretation could encompass collateral. Under this story, the payoff w includes the value of collateral posted by the entrepreneur, and then we can think of γ as representing any asymmetry between how the entrepreneur values the collateral and how the bank values it, plus the costs of selling the collateral. One could make a case that both interpretations came into play during the financial crisis. First, bankruptcy costs could have been perceived to have been higher, perhaps because of the notion that sorting out financial losses was going to be very difficult. Second, collateralizable assets may have been judged as being much more costly to sell.

A second factor that could reduce the supply of loans is a negative first-order stochastic dominance shift in the distribution of investment project returns $F(\cdot)$, i.e. an increase in $F(w)$ for each $w \in (0, \bar{w})$. From (39) and (40), this reduces the expected payoff to the bank given the loan interest rate and the verification cost γ, thus increasing the default premium. From (43) and (44), γ^* falls, given r, so $L(r)$ decreases for each r. Thus, given the policy setting (δ, μ), in a scarce interest-bearing assets equilibrium, we again get the qualitative effects in Figure 6. There is a reduction in lending and in the real interest rate, and an increase in default premia. Clearly, part of what was driving the financial crisis was a reduction in the perceived payoffs on certain classes of private assets, which we can capture in part here by this type of first-order stochastic dominance shift.

Finally, a factor in the financial crisis that this model can capture in a nice way is the effect of an increase in risk on the supply of private liquid assets, and the resulting general equilibrium implications. It is well-known that increases in risk can result in a reduction in lending, an increase in interest rate spreads, and an increase in defaults, in models with costly state verification. For an early analysis of these effects, see Williamson (1987). Christiano, Motto, and Rostagno (2009) measure the contribution of these "risk shocks" to business cycles, and find that they are very important.

A risk shock works essentially through the final term on the right-hand side of (39). A mean-preserving spread that amounts to a second-order stochastic dominance shift in the distribution $F(\cdot)$ can act to reduce the expected payoff to the bank, conditional on the loan interest rate. This occurs as the bank cares about the expected payoff to the investment project only in bankruptcy states - otherwise the payment to the bank is non-contingent. Thus, a mean preserving spread does nothing to increase the bank's returns in non-bankruptcy states, but lowers the expected return conditional on bankruptcy occurring.[4] Effectively, moving probability mass into the tails of the distribution of investment returns reduces the expected return to the bank in the event that an entrepreneur defaults, and this reduces the expected payoff on the loan given the loan interest rate. The ultimate effects are then similar to those of a first-order stochastic dominance shift, as discussed above. Given a policy (δ, μ), default premia increase and the quantity of loans and the real interest rate fall in an equilibrium with scarce interest-bearing assets.

[4] For the details, see Williamson (1987).

It is hard to argue that perceived risk was *not* an important factor during the financial crisis. A costly state verification setup shows how increased risk, in the context of non-contingent debt contracts, acts to contract credit, reduce rates of interest on safe assets, and increase interest rate spreads.

6.3.2 Monetary Policy: Conventional Open Market Operations

In the interest of brevity, it will be most useful in this subsection to consider an example with a specific utility function. Most of the results generalize, and in any event the main point of this section is to illustrate the possibilities.

Assume that $u(x) = 2x^{\frac{1}{2}}$ and, in the pre-financial crisis stage, that the condition for Proposition 3 holds, i.e. $L\left(\frac{1}{\beta}\right) > x^*$, where $x^* = 1$ given our utility function. Further, assume that $\sigma > 0$, so that the fiscal authority runs a deficit indefinitely. Then, the policy (δ, μ) is constrained, through non-passive fiscal policy, by (30) or

$$\delta = \frac{\beta(\mu - 1)\rho}{\sigma\mu^2}. \tag{47}$$

Then, assume that $\upsilon > 1 - \beta$, so that a pre-crisis optimal monetary policy is $\mu = \frac{\beta}{1-\upsilon}$ and, from (47),

$$\delta = \frac{(\beta - 1 + \upsilon)\rho(1 - \upsilon)}{\sigma\beta}.$$

Proposition 3 tells us this policy implies that $x = x^* = 1$ in monitored meetings in the DM, so that these trades are surplus maximizing, and then $\mu = \frac{\beta}{1-\upsilon}$ implies an inflation rate that taxes non-monitored exchange optimally.

Now, suppose that a financial crisis shock hits - some combination of shifts in $G(\cdot)$ and $F(\cdot)$ - and that this implies that credit market activity shuts down entirely, i.e. $L(r) = 0$ for all r. This is extreme, of course, but it helps to illustrate the ideas in a simple way, as we can now appeal to our results for the baseline model without private liquidity. Given a policy (δ, μ), from (15) we obtain a liquidity trap equilibrium when $\delta \geq \rho$, and in this equilibrium, from (14), we have

$$m = \frac{\beta\delta}{\mu}, \; x_n = x_m = \left(\frac{\beta}{\mu}\right)^2.$$

From (17), there is an equilibrium with plentiful interest-bearing assets if and only if

$$0 < \delta \leq \frac{\beta\rho}{\beta\rho + (1-\rho)\mu},$$

and, from (16), in this equilibrium we have

$$m = \frac{\beta\rho}{\mu}, \; x_n = \left(\frac{\beta}{\mu}\right)^2, \; x_m = x^* = 1.$$

Finally, from (21), an equilibrium with scarce interest-bearing assets exists if and only if

$$\frac{\beta\rho}{\beta\rho+(1-\rho)\mu}<\delta<\rho,$$

and, from (20) and (16), we get

$$m=\frac{\beta\rho}{\mu},\ x_n=\left(\frac{\beta}{\mu}\right)^2,\ x_m=\left[\frac{\beta\rho\left(\frac{1}{\delta}-1\right)}{(1-\rho)\mu}\right]^2,\ r=\frac{\rho\left(\frac{1}{\delta}-1\right)}{(1-\rho)\mu}. \tag{48}$$

Now, assume that $\sigma<\frac{\beta}{4}$, which guarantees that, given the constraint on the set of policies (47), there exist feasible policies that yield each of the three types of equilibria. Given this assumption and (47), we have the following:

1. If $\mu\leq 1$, then an equilibrium does not exist.
2. If $1<\mu\leq\mu_1$ or $\mu\geq\mu_2$ then an equilibrium with plentiful interest-bearing assets exists, where $\mu=\mu_1$ and $\mu=\mu_2$ are the two solutions to
$$\frac{\beta\rho}{\beta\rho+(1-\rho)\mu}=\frac{\beta(\mu-1)\rho}{\sigma\mu^2}.$$
3. If $\mu_1<\mu<\mu_3$ or $\mu_4<\mu<\mu_2$, then an equilibrium with scarce interest-bearing assets exists, where
$$\mu_3=\frac{\beta-\left(\beta^2-4\beta\sigma\right)^{\frac{1}{2}}}{2\sigma},\ \mu_4=\frac{\beta+\left(\beta^2-4\beta\sigma\right)^{\frac{1}{2}}}{2\sigma}.$$
4. If $\mu=\mu_3$ or $\mu=\mu_4$, then a liquidity trap equilibrium exists.
5. If $\mu_3<\mu<\mu_4$, then an equilibrium does not exist.

Then, for all of these equilibria, given μ we determine δ from (47). Then, given (47) and (48), we can solve for x_m and r in the scarce-interest-bearing-assets equilibrium, obtaining

$$x_m=\beta^2\left[\frac{\sigma\mu^2-\beta(\mu-1)\rho}{\beta(1-\rho)(\mu-1)\mu}\right]^2,\ r=\frac{\sigma\mu^2-\beta(\mu-1)\rho}{\beta(1-\rho)(\mu-1)\mu} \tag{49}$$

Then, for convenience assume that $\sigma>\rho\beta$, which implies that x_m and r are decreasing in μ, in general, and from (49) x_m and r are strictly decreasing in μ in the scarce interest-bearing assets equilibrium.

Now, given our assumptions, once the financial crisis occurs, there are four cases to be concerned with. First, if $1-\beta<\upsilon\leq 1-\frac{\beta}{\mu_1}$ or $\upsilon\geq 1-\frac{\beta}{\mu_2}$, then the optimal pre-crisis policy is also optimal post-crisis, as this policy still implies an equilibrium with plentiful interest-bearing assets. The crisis produces a drop in output in the CM, as private investment projects are no longer being financed, but the quantities exchanged in the DM remain unchanged. In this case, while the crisis disrupts credit, it does not disrupt exchange, as there is sufficient government-supplied liquidity to support efficient exchange, correcting for the inefficiency associated with socially-useless exchange using currency.

Second, suppose that $1 - \frac{\beta}{\mu_4} < \upsilon \leq 1 - \frac{\beta}{\mu_2}$. In this case, if the pre-crisis policy is maintained, then the crisis will now disrupt monitored exchange in the DM. The decrease in private lending produces a scarcity of interest-bearing assets, reflected in a lower gross real interest rate r. Further, the optimal policy response, from (36)-(38), and given that r is decreasing in μ, is a reduction in μ. We cannot say whether δ will be higher or lower in equilibrium, but r must increase due to the change in policy. The equilibrium under the post-crisis optimal policy will either have scarce interest-bearing assets or a liquidity trap. Key results here are that the crisis implies a lower optimal inflation rate and a higher real interest rate, as this reflects a mitigation of the asset scarcity problem.

Third, suppose that $1 - \frac{\beta}{\mu_3} < \upsilon < 1 - \frac{\beta}{\mu_4}$. Then the pre-crisis policy cannot be maintained, as it is no longer feasible. It is possible that the post-crisis optimal policy has $\mu = \mu_4$ and $\delta = \rho$, so that there is a liquidity trap. The real interest rate has fallen from its pre-crisis level but the inflation rate is actually higher, due to the fact that the liquidity trap makes it infeasible to achieve a lower inflation rate, which would otherwise be optimal. It could be optimal in these circumstances to set policy such that $\mu = \mu_3$ and $\delta = \rho$ to achieve a liquidity trap equilibrium with a lower inflation rate than was the case pre-crisis.

Fourth, in the case where $1 - \frac{\beta}{\mu_1} < \upsilon \leq 1 - \frac{\beta}{\mu_3}$, maintaining the pre-crisis policy will imply that there is an equilibrium with scarce interest-bearing assets post-crisis, so that the real interest rate falls. Here, welfare will increase with the choice of lower μ and lower δ, which implies that the real interest rate increases and the inflation rate falls, and the optimal policy (from (36)-(38)) will imply an equilibrium with scarce interest-bearing assets.

Thus, one conclusion from this example is that the increase in credit market frictions brought on by the financial crisis can cause disruptions in the exchange process. A common view of the recent financial crisis is that it differed from historical episodes of financial stress because of the *lack* of disruption in payments. During the National Banking era in the United States (1863-1913) there were recurrent banking panics, and there were panics and massive bank failures in the U.S. during the Great Depression, as is well-known. In all of these episodes, a key negative effect on real activity seems to have come from a disruption in retail payments activity. During the recent financial crisis, there were no such payments disruptions. Some small banks failed, but in an orderly fashion that had no implications for retail payments.

While Gorton (2010) makes the case that there were disruptions in the "shadow banking" sector akin to traditional bank runs during the recent financial crisis, the mechanism at work in our model is much different. Here, an increase in financial frictions produces a shortage of liquid assets, and the government can mitigate this shortage. However, optimal government intervention does not proceed in the traditional way that, for example, Friedman and Schwartz (1963) envisioned, which essentially involves flooding the market with currency to substitute for missing privately-supplied media of exchange. Here, the liquid assets that are in short supply are interest-bearing assets, and if monetary policy acts to mitigate this shortage, that will tend to increase the real interest rate and reduce the inflation rate. This certainly is not a typical Keynesian policy prescription, which would typically call for a reduction in the real interest rate in circumstances like these. Further, the shortage of interest-bearing assets is mitigated, under some circumstances, through open market sales (lower δ) rather than open market purchases.

Another interesting feature of the example is that the occurrence of a financial crisis may imply an optimal monetary policy that puts the economy in a liquidity trap equilibrium. There is a sense in which the liquidity trap constrains policy, but the liquidity trap is unavoidable. It is also not a "trap," as the monetary authority could get out of the trap if it wanted to, but it may not be optimal to do so.

6.3.3 Private Asset Purchases by the Central Bank

The Federal Reserve Act, Section 13(3), gives the Fed substantial powers to purchase assets of its own choosing, and those powers were used extensively during the recent financial crisis and after. In this section we will focus on central bank asset purchases that, in the model, look much like the purchases of over $1 trillion in mortgage-backed securities, executed by the Fed between February 2009 and June 2010. This type of asset purchase program is sometimes referred to as "quantitative easing," and the particular one we have in mind is sometimes called "QE1."

One approach to capturing central bank purchases of private assets in our model, is to suppose that the central bank has access to the same verification technology as the private sector, and avails itself of the same efficient debt contracts as do private banks in lending to entrepreneurs. Thus, we will assume that central bankers are no more (and no less) capable than private sector bankers. Assume that the central bank lends to a mass k of private entrepreneurs each period, and makes these loans on the same terms as would the private sector, i.e. if the gross real interest rate is r, each private sector loan made by the central bank earns an expected gross return of r. The central bank finances its lending by issuing $\phi_t E_t$ units of reserve balances (in real terms) in period t, where $\phi_t E_t = k$ for each t. In each period, the central bank uses the returns on its loan portfolio to pay interest on reserves at the real gross rate r. Provided that $k \leq L(r)$ (lending by the central bank does not exceed the quantity of lending in the absence of this policy intervention), this policy will have no effect on prices or quantities. The quantity of assets a in a stationary equilibrium will be

$$a = m\left(\frac{1}{\delta} - 1\right) + L(r) - k + \phi_t E_t = m\left(\frac{1}{\delta} - 1\right) + L(r)$$

Therefore, given our preceding analysis for the private liquidity case, if the real interest rate r is an equilibrium interest rate in the absence of central bank lending, then if $0 < k \leq L(r)$ and the central bank sets an interest rate on reserves equal to r, the equilibrium allocation will be identical to what was achieved without central bank lending. The central bank simply adds another layer of redundant intermediation, there is an expansion in the stock of outside money, and there is no effect on prices.

Things are different, however, if the central bank lends on better terms than does the private sector. Suppose, for example, that the private sector makes loans to $L(r)$ entrepreneurs, with each loan yielding an expected gross real return r to private banks. The central bank then lends $L(\bar{r}) - L(r)$ to marginal borrowers, offering efficient loan contracts that each yield an expected gross return $\bar{r}$ to the central bank, where $\bar{r} < r$. This loan portfolio is financed with reserves that earn a gross interest rate r, which is set by the central bank. Now, the quantity of assets held by banks in equilibrium is given by

$$a = m\left(\frac{1}{\delta} - 1\right) + L(\bar{r}),$$

and $\bar{r}$ is another policy instrument for the central bank. The central bank will now suffer a loss each period on its lending activities, equal to $(r - \bar{r})\left[L(\bar{r}) - L(r)\right]$, which we will assume is financed by lump-sum taxation on buyers in the CM.

What is the effect of this policy? First, if parameters are such that the economy is initially in a plentiful-interest-bearing-assets equilibrium, and if (δ, μ) remains the same, then there is no effect, except for a redistribution from buyers to the group of entrepreneurs who would otherwise not be funded. With plentiful assets, the government lending program just adds to the stock of liquid intermediated assets, and does not affect exchange. Credit is allocated in a different way, however, and buyers suffer as a result (because they are taxed to make up for the central bank losses). Second, suppose that the economy is initially in an equilibrium with scarce liquid assets. Then, for a given (δ, μ), the real interest rate will be higher. There is a beneficial effect, in that there is a larger supply of liquid assets, and exchange is more efficient in monitored transactions during the DM. However, entrepreneurs who borrow privately suffer relative to those who borrow cheaply from the central bank, and buyers also pick up the tab for the losses on the central bank portfolio.

Therefore our model suggests that, at best, the Fed's QE1 program had no effect. If it did have any effects, then this would be because the central bank was offering better terms to borrowers than they would receive in the private sector, thus implying that the Fed was reallocating credit and redistributing wealth through the program.

7 Conclusion

Fundamental to New Monetarist economics is the idea that the explicit roles played by particular assets in transactions, and how assets are intermediated, are critical to understanding the interaction among financial and monetary phenomena, quantities, and prices. This paper brings together some elements of recently-developed monetary theory, and received theories of financial intermediation, to provide some new insights concerning monetary policy and the financial crisis.

In the model, there exists a permanent nonneutrality of money - an illiquidity effect - whereby an open market purchase of government bonds reduces the supply of liquid assets available in a particular class of transactions, thus reducing the real interest rate and expanding lending. An equilibrium with a liquidity trap can exist under non-Friedman-rule monetary policies. In the model, a non-passive regime for fiscal policy and costs of operating the currency system yield a departure from the Friedman rule at the optimum.

Under shocks that replicate some qualitative observations related to the financial crisis, there can be disruptive effects on exchange, as these shocks make liquid interest-bearing assets more scarce. It can be optimal, in response to such shocks, for the central bank to use conventional open market operations to increase safe real rates of interest and reduce inflation. If the central bank purchases private assets, this is at best irrelevant, changing no prices or quantities, but such purchases can also reallocate credit and redistribute wealth.

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9 Appendix (Not for Publication)

Proof. [Proof of Propositions 1(i), 2(i), and 3(i)] In a liquidity trap equilibrium with private liquidity, instead of (14) we have

$$\frac{\beta}{\mu}u'\left[\frac{\beta}{\mu}\left(\frac{m}{\delta}+L\left(\frac{1}{\mu}\right)\right)\right]=1, \tag{50}$$

which solves for m. Then, a liquidity trap equilibrium exists if and only if the solution m to (50) implies that there is sufficient currency to finance the purchases of non-monitored depositors, with some excess cash held by banks as reserves. Therefore,

$$\frac{1}{\delta}\leq\frac{1}{\rho}-\frac{L\left(\frac{1}{\mu}\right)}{m(\mu)} \tag{51}$$

Thus, if

$$x^{*}-L\left(\frac{1}{\beta}\right)\geq 0, \tag{52}$$

holds, then (51) holds for for $\delta \geq \delta_a$ and $\mu \in (\beta, \mu_b(\delta)]$. However (51) does not hold for $0 < \delta < \delta_a$ if (52) holds. If $\delta < 0$ and (52) holds then (51) holds for $\mu \in (\mu_a, \mu_b(\delta)]$. Now if (52) does not hold, then from (51) an equilibrium of this type cannot exist for $\delta > 0$. If $\delta_a \leq \delta < 0$, and (52) does not hold, then (51) holds for $\mu \in (\beta, \mu_b(\delta)]$. ■

Proof. [Proof of Propositions 1(ii), 2(ii), and 3(ii)] In an equilibrium with plentiful interest-bearing assets, the real quantity of currency, m, is determined by (16). A necessary and sufficient condition for existence of this equilibrium is that the supply of interest-bearing assets at the gross real interest rate $\frac{1}{\beta}$ be sufficient to support surplus maximizing exchange in monitored DM meetings, or $a \geq (1-\rho)x^*$. From (46), this gives

$$m\left(\frac{1}{\delta}-1\right) \geq (1-\rho)x^* - L\left(\frac{1}{\beta}\right). \tag{53}$$

Thus, given (δ, μ) with $\delta \in (-\infty, \infty)$, $\delta \neq 0$, and $\mu > \beta$, if (17) is satisfied given the m that solves (16), then this equilibrium exists. Now, first consider the case where

$$(1-\rho)x^* - L\left(\frac{1}{\beta}\right) > 0. \tag{54}$$

Then, if $\delta < 0$ or $\delta > \delta_a$, (53) does not hold so an equilibrium does not exist; however (53) holds if $0 < \delta \leq \delta_a$ for $\mu \in (\beta, \mu_c(\delta)]$, so an equilibrium exists under those conditions. Next, consider the case where

$$(1-\rho)x^* < L\left(\frac{1}{\beta}\right) < x^*. \tag{55}$$

Now (53) does not hold for $\delta < 0$, so an equilibrium does not exist; but if $0 < \delta \leq \delta_a$ then (53) holds for any $\mu > \beta$, in which case an equilibrium exists. As well, if $\delta > \delta_a$ and $\mu \geq \mu_c(\delta)$, then (17) holds and an equilibrium exists. Finally, consider the case where (52) does not hold. Then, (53) holds if $\delta > 0$ or $\delta < \delta_a$ for all $\mu > \beta$, so an equilibrium exists in those cases. As well, if $\delta_a \leq \delta < 0$ and $\mu \geq \mu_c(\delta)$, then (53) holds and an equilibrium exists. Otherwise, (53) does not hold, in which case an equilibrium with plentiful interest-bearing assets does not exist. ■

Proof. [Proof of Propositions 1(iii), 2(iii), and 3(iii)] An equilibrium with scarce interest-bearing assets consists of (m, r, a) satisfying (16), (19), and (46). First, recall that $m = m(\mu)$ denotes the solution to (16). Since $-\frac{xu''(x)}{u'(x)} < 1$, $m(\mu)$ is a strictly decreasing function, and is also continuous for $\mu \geq \beta$, with $m(\beta) = \rho x^*$. The key characteristic of this equilibrium is that the solution satisfies

$$\frac{1}{\mu} < r < \frac{1}{\beta}, \tag{56}$$

but given that $-\frac{xu''(x)}{u'(x)} < 1$, if (19), and (46) yield a solution for a and r given m that satisfies (56), then that solution is unique. Thus, determining the conditions for existence here amounts to finding the policies (δ, μ) that imply a solution for (m, r, a) satisfying (56). From (16), (19), and (46), then, (56) is satisfied if and only if the following two conditions hold.

$$m\left(\frac{1}{\delta}-1\right) < (1-\rho)x^* - L\left(\frac{1}{\beta}\right) \tag{57}$$

$$\frac{1}{\delta} > \frac{1}{\rho} - \frac{L\left(\frac{1}{\mu}\right)}{m(\mu)} \tag{58}$$

If (54) holds, then (57) and (58) hold if and only if $0 < \delta \leq \delta_a$ and $\mu > \mu_c(\delta)$, or $\delta > \delta_a$ and $\mu > \mu_b(\delta)$, or $\delta < 0$ and $\mu > \mu_b(\delta)$. However, if (55) holds, then (57) and (58) hold if and only if $\delta > \delta_a$ and $\mu \in (\mu_b(\delta), \mu_c(\delta))$, and for $\delta < 0$ and $\mu \in (\mu_b(\delta), \mu_c(\delta))$. Finally, if (52) does not hold, then (57) and (58) cannot both hold if $\delta > 0$ or $\delta \leq \delta_a$, but if $\delta_a < \delta < 0$, then (57) and (58) hold if and only if $\mu \in (\mu_b(\delta), \mu_c(\delta))$. ■