

Self-Insurance vs. Self-Financing: A Welfare Analysis of the Persistence of Shocks

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September 8, 2009

Abstract

We examine the welfare cost of market incompleteness in a generalized Bewley model. The idiosyncratic risk in our economy takes the form of entrepreneurial productivity shocks. There are two dimensions of market incompleteness in our framework. Firstly, in the Bewley tradition, only a limited set of instruments for consumption smoothing are available. Secondly, the capital rental market may not function efficiently because of imperfect enforceability of contracts. We first show that the frictions in the capital rental market can inflict substantial welfare costs. We then quantify how the persistence of idiosyncratic shocks affects welfare. In conventional Bewley models, the persistence of shocks adversely affects welfare. With imperfect capital rental markets, however, the persistence of individual productivity enables constrained entrepreneurs to overcome the capital market frictions over time with self-financing. With intermediate levels of frictions in the capital market, welfare costs of market incompleteness have a U shape against the persistence of idiosyncratic shocks. The right arm of the U reflects the difficulty of self-insurance against very persistent shocks; and the left arm, the difficulty of overcoming capital market frictions through self-financing when entrepreneurial opportunities are short-lived.

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1 Introduction

How costly is market incompleteness? In this paper, we answer this question with a generalized Bewley model.

Let us first be clear about what we mean by Bewley models. We follow Ljungqvist and Sargent (2004) and use the term “Bewley models” to refer to the large (and still expanding) class of incomplete-market heterogeneous-agent models. Bewley models share a common structure: The model builder exogenously limits the set of instruments that individuals can use to smooth their consumption against idiosyncratic income fluctuations. The unavailability of explicit consumption insurance is often justified perfunctorily on the grounds of enforcement or information problems. On the production side, however, all factor and output markets operate efficiently, and a representative firm is assumed. One can easily imagine the same enforcement or information problems that disallow consumption insurance contracts disrupting the production side of the economy as well—e.g., interfering with the allocation of capital to its most productive use.

We develop this idea and introduce frictions in the capital rental market, which arise from imperfect enforceability of contracts. We build a model of individual-specific technologies (entrepreneurship), where idiosyncratic risk takes the form of shocks to individuals’ entrepreneurial productivity. Such a model has proven successful in explaining key features of the wealth inequality in the US (Quadrini, 2000; Cagetti and De Nardi, 2006). Now we have two distinct dimensions of market incompleteness in the model: missing consumption insurance—as in conventional Bewley models—and frictions in the capital rental market. Our model nests the traditional Bewley models: With perfectly-functioning capital rental markets, our economy is isomorphic to the one studied by Aiyagari (1994).

We now re-examine the welfare cost of market incompleteness using our model. One well-known result from the large literature building on Bewley (1980) is that the welfare cost of market incompleteness can be substantial if the uninsurable idiosyncratic shocks are very persistent (Deaton, 1991; Huggett, 1993; Aiyagari, 1994). In the absence of contracts contingent on their income risk, individuals accumulate a buffer stock of savings to smooth their consumption. This behavior is known as *self-insurance*. It is harder to self-insure against more persistent shocks, and hence the welfare cost of missing consumption insurance increases with the persistence of idiosyncratic shocks.

One result from our generalized Bewley model is that the welfare cost from the frictions in the capital rental market can be substantially higher than that from missing consumption insurance. More important, the welfare cost of capital market frictions is lower, not higher, when the idiosyncratic shocks are more persistent. The reason is as

follows. With frictions in the capital rental market, entrepreneurs with profitable projects need to self-finance their production, since they can not raise enough external financing. When the entrepreneurial productivity shocks are persistent, poor entrepreneurs with profitable projects will find it worthwhile to save up so that they can self-finance their production and earn higher income in the future. Also, the persistent shock process implies that, probabilistically speaking, many of these entrepreneurs will eventually succeed. With persistent productivity shocks, *self-financing* is an effective substitute for external financing, thereby alleviating the associated welfare cost. On the other hand, when the shocks are not very persistent, poor entrepreneurs with good projects are more likely to lose their profitable production opportunities before they can save up enough capital for self-financing. This low probability of eventual success deters them from attempting to overcome the capital market frictions in the first place. Without self-financing effectively substituting for the imperfect capital market, the welfare cost is high.

In our model, the overall welfare cost of market incompleteness comes from missing consumption insurance and from imperfect capital markets. The former channel manifests itself primarily through the dispersion in consumption. When shocks are made more persistent—holding their unconditional variance unchanged, self-insurance becomes less effective and the variance of individual consumption increases. With well-functioning capital rental markets, the mean of individual consumption increases only modestly with the persistence of shocks, as individuals’ precautionary saving drives up the aggregate capital stock in the stationary equilibrium.¹ In such cases, the overall welfare cost of market incompleteness is *increasing* in the persistence of shocks. With frictions in the capital rental market, however, the persistence of shocks facilitates self-financing by entrepreneurs with profitable production opportunities, and hence increases the aggregate output and consumption. With enough frictions in the capital rental market, this increase in the average consumption can more than offset the welfare loss from the higher consumption variance, and the overall welfare cost of market incompleteness is *decreasing* in the persistence of shocks—a result hitherto not obtained with conventional Bewley models.

With intermediate levels of frictions in the capital rental market, the overall welfare cost of market incompleteness is U-shaped when plotted against the persistence of idiosyncratic shocks. The right (and upward) arm of the U reflects the difficulty of self-insurance against very persistent shocks. The left (and downward) arm, the difficulty of overcoming capital market frictions through self-financing when individuals’ entrepreneurial opportu-

¹This is the celebrated capital over-accumulation result of Aiyagari (1994). See Angeletos (2007) for more on this.

nities are short-lived.

The early debates on the welfare cost of uninsurable idiosyncratic risk have given rise to a large literature on the persistence of individual income processes.² Our analysis contributes to this literature by suggesting that one should also think about the sources of income fluctuations: The persistence of shocks has different welfare implications depending on whether income itself is the shock or income has an endogenous component that responds to underlying shocks on ability or opportunities.

Quantitative Framework We incorporate entrepreneurship and capital market frictions into an otherwise-standard neoclassical model with incomplete markets. In our model, individuals choose whether to operate an individual-specific technology—become entrepreneurs—or to supply labor for a wage in each period. Individuals differ from one another in their productivity as an entrepreneur and in their wealth. The former is driven by a stochastic process (idiosyncratic risk), while the latter is endogenously determined by forward-looking saving decisions. We model frictions in capital rental markets as arising from imperfect enforceability of rental contracts, which gives rise to collateral constraints. These constraints hinder efficient allocation of capital across entrepreneurs and distort entry into entrepreneurship. As in conventional Bewley models, a risk-free bond is the only asset available for consumption smoothing.

For our calibration, we rely for the most part on the standard parameter values in the literature. The parameters governing the entrepreneurial productivity distribution are more specific to our model, and we pin them down using the US data on the concentration of entrepreneurial income.

Starting from the benchmark with complete markets, we quantify the welfare costs of market incompleteness by first shutting down the consumption insurance market. We then vary the persistence of the idiosyncratic shocks while holding constant their unconditional variance. We report the welfare difference between the resulting stationary equilibria and the complete-market benchmark, in units of permanent consumption compensation. Because the individuals in our model are ex-ante identical, it is natural to use the perfectly-egalitarian measure of economy-wide welfare.

Next, we introduce collateral constraints in the capital rental market. Explicit consumption insurance is still ruled out. Again, we vary the persistence of idiosyncratic shocks, and then compute the additional welfare cost for the corresponding stationary

²See Guvenen (2007, 2009) and the references therein for an overview of the ongoing debates on restricted income profiles (RIP) vs. heterogeneous income profiles (HIP).

equilibria, thereby obtaining a decomposition of the overall welfare cost of market incompleteness.

Related Literature Given the vastness of the literature built on Bewley models and also the purpose of this paper, we do not attempt to review all contributions in this area. We instead refer interested readers to the excellent survey of Heathcote et al. (2009).

In the early Bewley literature, the source of heterogeneity (and hence inequality) is invariably labor income, which is assumed to be purely exogenous. Recently, researchers have recognized that individual income dynamics contains endogenous components reflecting individual decisions on occupation, labor supply, job search, and the accumulation of assets and human capital. In this context, we highlight the occupation choices in our model, which depend not only on individuals' shock realizations but also on their saving decisions in the past and the functioning of capital markets in the economy.

In traditional Bewley models, individuals have access only to risk-free saving and borrowing for smoothing their consumption. This clearly understates the range of formal and informal risk-sharing instruments available to people in reality. On the other hand, one can argue that these models abstract from other (arguably important) idiosyncratic risk that individuals face over the course of their lifetime: e.g., health shocks and family events. In this respect, our model is closely aligned with the conventional Bewley models: We focus on the source of risk (entrepreneurial productivity) and the instrument of insurance (buffer stock of risk-free assets) most relevant for the question of interest. In particular, while most work in the literature is built on individual labor productivity shocks, we abstract from them. As a result, the left tail of our income distribution is not consistent with data. While this issue can be easily handled by a straightforward extension, it is not at all important for the main point that we make in this paper.

The papers that are most closely related to ours are Quadrini (2000) and Cagetti and De Nardi (2006). They incorporate financial frictions into models with entrepreneurship, and show that these elements explain the empirical wealth distribution. Intuitively, if there are financial frictions, highly-talented entrepreneurs will self-finance and hold a large ownership stake in their own businesses, which translates into a fat right tail of the wealth distribution. We build on their analysis, and study the welfare cost of market incompleteness in this class of models.³

³We use the framework developed in Buera and Shin (2008) to carry out our analysis. While the current paper focusses on the welfare analysis of stationary equilibria, Buera and Shin (2008) studies the transition dynamics of the macroeconomy following a large-scale reform that creates the need for reallocation of resources among heterogeneous producers, with a particular emphasis on the role of production-side

Yet another paper deserves further discussion. Heathcote et al. (2008) show that an increase in idiosyncratic labor productivity risk may have a positive welfare effect in an incomplete-market model with endogenous labor supply. While larger uncertainty increases the welfare cost from missing consumption insurance, it also presents an “opportunity” to raise aggregate productivity by concentrating market work among more productive workers. The positive welfare effect is more likely to prevail when the idiosyncratic labor productivity is less persistent and hence more self-insurable. That is, welfare is decreasing in the persistence of shocks, as in traditional Bewley models. In this regard, our result offers a novel and somewhat contrarian insight on the welfare effect of the persistence of shocks.

2 Model

We propose a model with individual-specific technologies and imperfect capital markets. Individuals are heterogeneous with respect to their entrepreneurial productivity and wealth. In each period, individuals choose either to operate an individual-specific technology—to become entrepreneurs—or to work for a wage. Imperfection in the capital rental market is modeled with a collateral constraint that is proportional to an individual’s financial wealth. As in the conventional Bewley models, there is no explicit consumption insurance, and individuals can only use a risk-free asset to self-insure against idiosyncratic risk.

Heterogeneity and Demographics Individuals live indefinitely, and are heterogeneous with respect to their wealth a and their entrepreneurial productivity $e \in \mathcal{E}$, with the former being chosen endogenously by forward-looking saving decisions. An individual’s entrepreneurial productivity follows a stochastic process. One can think of the entrepreneurial productivity shock as an arrival of a new technology making previous production processes less profitable or as a shift in relative demand.

We denote by $\mu(e)$ the measure of individuals with entrepreneurial productivity e in the invariant distribution. We denote by $G_t(e, a)$ the cumulative density function for the joint distribution of entrepreneurial productivity and wealth at the beginning of period t . Naturally, $G_t(a|e)$ is the associated c.d.f. of wealth for a given entrepreneurial productivity e . The population size of the economy is normalized to one, and there is no population growth.

financial market frictions in delaying such reallocation.

Preferences Individuals discount their future utility using the same discount factor β . The preferences over contingent plans for the consumption sequence from the point of view of an individual in period t are represented by the following expected utility:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} u(c_s).$$

Technologies In any given period, individuals can choose either to work for a wage or to operate an individual-specific technology. We label the latter option as entrepreneurship. We assume that an entrepreneur with productivity e who uses k units of capital and hires l units of labor produces according to a production function $f(e, k, l)$, which is assumed to be strictly increasing in all arguments, and strictly concave in capital and labor, with $f(0, k, l) = 0$.

Financial Markets A risk-free one-period bond and physical capital are the only financial and real assets in the economy. A perfectly-competitive financial intermediary sells bonds backed by the physical capital stock to households, and rents out capital to entrepreneurs. Individuals can also issue bonds (private IOUs) to other individuals subject to quantity restrictions. This quantity limit can be zero (no borrowing allowed for intertemporal consumption smoothing), and actually will be assumed to be zero in our analysis. It is assumed that all the bonds are repaid with probability one, and that their payoff is not state-contingent, making them risk-free. Furthermore, bonds issued by the competitive intermediary and those by individuals are interchangeable. In equilibrium, the net demand for bonds by individuals is equal to the stock of physical capital. The risk-free return on bonds—i.e. the risk-free interest rate in the economy—is r . The zero-profit condition of the intermediary implies that the rental cost of capital is $r + \delta$, where δ is the depreciation rate.

We assume that entrepreneurs' capital rental (k) is limited by a collateral constraint $k \leq \lambda a$, where a is individual financial wealth and λ measures the degree of credit frictions, with $\lambda = +\infty$ corresponding to perfect capital rental markets, and $\lambda = 1$ to financial autarky where all capital needs to be self-financed by entrepreneurs. The same λ applies to everyone in a given economy.

Our specification captures the common prediction from models of limited contract enforcement: The amount of credit is limited by entrepreneurs' wealth. This specification has been widely used in the literature on financial frictions and entrepreneurship (Evans and Jovanovic, 1989), and also in the literature on credit frictions and business cycles (Bernanke et al., 1999; Kiyotaki and Moore, 1997).

Our collateral constraint can be derived from the following limited enforcement problem. Consider an individual with financial wealth a (deposited in the financial intermediary) at the beginning of a period. Assume that he rents k units of capital. Then he may choose to abscond with a fraction $(1/\lambda)$ of the rented capital. The only punishment is that he will lose his financial wealth a deposited with the intermediary. In particular, he will not be excluded from any economic activities in the future. In fact, he is allowed to instantaneously deposit the stolen capital k/λ and continue on as a worker or an entrepreneur. Note that λ in this context measures the degree of capital rental contract enforcement, with $\lambda = +\infty$ corresponding to perfect enforcement and $\lambda = 1$ to no enforcement. In the equilibrium, the financial intermediary will rent capital only to the extent that no entrepreneur will renege on the rental contract, which implies a collateral constraint $k/\lambda \leq a$ or $k \leq \lambda a$.

In this paper we focus on within-period borrowing, or capital rental, for production purposes. We do not allow borrowing for intertemporal consumption smoothing in our model, which translates into $a \geq 0$. This no-borrowing constraint (in the language of Bewley models) will only bind for individuals who choose to be workers, and has no direct bearing on the behavior of entrepreneurs, who will need to hold financial assets to overcome the collateral constraint.

By not allowing individuals to borrow for consumption smoothing, we are obtaining the upper bound on the welfare cost from missing consumption insurance. In this sense, we are stacking the cards against the welfare cost from capital rental market frictions prevailing.

Individuals' Problem The recursive representation of the problem of an individual is given by the following Bellman equation:

$$\begin{aligned} v(a, e) &= \max_{c, a' \geq 0} u(c) + \beta \mathbb{E}[v(a', e') | e] \\ \text{s.t. } c + a' &\leq \max\{w, \pi(a, e; w, r)\} + (1 + r)a. \end{aligned} \tag{1}$$

where the equilibrium wage w and interest rate r are given, and $\pi(a, e; w, r)$ is the profit from operating an individual technology. This indirect profit function is defined as:

$$\pi(a, e; w, r) = \max_{l, k \leq \lambda a} \{f(e, k, l) - wl - (\delta + r)k\}.$$

The input demand functions are denoted by $l(a, e; w, r)$ and $k(a, e; w, r)$, and the collateral constraint ($k \leq \lambda a$) is taken into account.

The max operator in the budget constraint stands for the occupation choice. An individual with current wealth a and entrepreneurial productivity e will choose to be an entrepreneur only if profits as an entrepreneur, $\pi(a, e; w, r)$, exceed labor income as a wage earner, w . More formally, an individual with productivity e decides to be entrepreneurs if his current wealth a is higher than the threshold wealth $\underline{a}(e)$, where $\underline{a}(e)$ solves:

$$\pi(\underline{a}(e), e; w, r) = w.$$

Intuitively, individuals with a given entrepreneurial productivity will choose to become entrepreneurs only if they are wealthy enough to overcome the collateral constraint and run their businesses at a profitable scale. Similarly, individuals of a given wealth level choose to become entrepreneurs only if their entrepreneurial productivity is high enough.

As we make the simplifying assumption that individuals are equally productive as workers, the occupation choice serves as an avenue of insurance from the downside risk of the stochastic entrepreneurial productivity. This feature is in line with recent modifications introduced to conventional Bewley models, such as endogenous labor supply decisions (Low, 2005; Heathcote et al., 2008).

Stationary Competitive Equilibrium A stationary competitive equilibrium consists of the invariant distribution of entrepreneurial productivity and wealth $G_\infty(e, a)$, policy functions $\{c(e, a), a'(e, a), l(e, a), k(e, a)\}$, and prices $\{w, r\}$ such that:

1. Given $\{w, r\}$, e , and a , $\{c(e, a), a'(e, a), l(e, a), k(e, a)\}$ solve the individuals' problem in (1);
2. The labor and capital markets clear, which by Walras' law implies goods market clearing as well:

$$\sum_{e \in \mathcal{E}} \mu(e) \left[\int_{\underline{a}(e, w, r)}^{\infty} l(a, e; w, r) G_\infty(da|e) - G_\infty(\underline{a}(e, w, r)|e) \right] = 0, \quad (\text{Labor})$$

$$\sum_{e \in \mathcal{E}} \mu(e) \left[\int_{\underline{a}(e, w, r)}^{\infty} k(a, e; w, r) G_\infty(da|e) - \int_0^{\infty} a G_\infty(da|e) \right] = 0; \quad (\text{Capital})$$

3. The invariant distribution of ability and wealth $G_\infty(e, a)$ satisfies the equilibrium mapping:

$$G_\infty(a|e) = \frac{1}{\mu(e)} \sum_{\hat{e} \in \mathcal{E}} \mu(e|\hat{e}) \mu(\hat{e}) \int_{u \leq a} \int_{a'(\hat{e}, v) = u} G_\infty(dv|\hat{e}) du.$$

3 Quantitative Exploration

3.1 Calibration

We first describe the parametrization of the model, and then discuss the calibration of the parameters that are held constant as we vary the persistence of the idiosyncratic shock. We consider different degrees of shock persistence primarily because this parameter is central to understanding the relative effectiveness of self-insurance and self-financing in alleviating the welfare cost of market incompleteness. In addition, this parameter is hard to pin down in empirical work, and a lot of controversy surrounds its estimates. To this date, researchers seem to lack the high-quality panel data on individual income that could provide a definitive answer.

For the sake of clarity, we choose a parsimonious parametrization that follows as much as possible the standard practices in the literature.

We choose a period utility function of the constant relative risk aversion form:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}.$$

We assume that an entrepreneur with productivity level e who hires k units of capital and l units of labor produces according to the following production function:

$$f(e, k, l) = e (k^\alpha l^{1-\alpha})^{1-\nu},$$

where ν is the share of output going to the entrepreneur, and $1 - \nu$ is the span-of-control parameter (Lucas, 1978). Accordingly, $1 - \nu$ represents the share of output going to the variable factors. Out of this, fraction α goes to capital, and $1 - \alpha$ goes to labor.

The entrepreneurial productivity e is assumed to follow a discretized version of an autoregressive process with normal innovations, $\log e' = \rho \log e + \varepsilon$, with $\varepsilon \sim N(0, \sigma^2)$.

We now need to specify six parameter values: technological parameters α , ν , and the depreciation rate δ ; the parameter describing the dispersion of entrepreneurial productivity σ ; the subjective discount factor β and the coefficient of relative risk aversion γ . As we describe below, we also need to pick a benchmark value of ρ in order to calibrate β .

We let $\gamma = 1.5$ following the standard practice. The one-year depreciation rate is set at $\delta = 0.06$. We impose $\alpha(1 - \nu) = 0.30$ to match the aggregate share of capital.

We are thus left with three parameters (ν , σ , and β). We choose them so that the model economy with perfect capital rental markets ($\lambda = \infty$) matches relevant moments in the US data on the income concentration in the overall population and among entrepreneurs, and the interest rate.

With the target annual real interest rate of four per cent, we calibrate ν and $\sigma^2/(1 - \rho^2)$, the unconditional variance of the log entrepreneurial productivity, to match the income concentration among the top five per cent of earners in the population and the fraction of total entrepreneurial income in the hands of the top ten percent of entrepreneurs. In particular, we can infer ν from the fraction of total income accounted for by the top five per cent of earners in the population: Top earners are mostly entrepreneurs both in the data and in our model, and ν controls the share of income going to the entrepreneurial input. Given the span-of-control parameter $1 - \nu$, the dispersion of the entrepreneurial productivity can be chosen to match the right tail of the income distribution among entrepreneurs.

In order to calibrate the discount rate, we pick $\rho = 0.98$ as a benchmark value. This choice of ρ is consistent with the estimated persistence of individual income processes in the majority of empirical work. Note that, in our analysis below, we vary ρ while holding other parameter values constant to analyze the welfare of the corresponding stationary equilibria. Given ρ , we choose β so that saving decisions in the model are consistent with an interest rate of four per cent.⁴

3.2 Results

We now explore how the welfare cost of market incompleteness in our generalized Bewley model changes with the persistence of the idiosyncratic shock. We compare the welfare of the complete-market benchmark with that of three incomplete-market counterparts. In the first incomplete-market economy, we shut down the market for consumption insurance, and let individuals use only risk-free bonds for self-insurance. The capital rental market still functions perfectly. This is our model without collateral constraints ($\lambda = \infty$), which is isomorphic to the one studied by Aiyagari (1994). In the second incomplete-market economy, we now introduce frictions in the capital rental market as well, in the form of collateral constraints ($\lambda \ll \infty$). For this economy, we assume $\lambda = 7.5$, which translates into rather mild frictions in the capital rental market. Our third incomplete-market economy is qualitatively similar to the second one, except that it has more severe financial frictions in the capital rental market, with $\lambda = 1.5$. These λ 's are consistent with the variation in the ratios of external financing to GDP observed in economies in different stages of development. In all of our three incomplete-market economies, no explicit

⁴We also considered a calibration strategy where β is re-calibrated for each value of ρ . This strategy gave us a good sense of how sensitive our calibration is to the persistence of idiosyncratic shocks (not much). In the end, we chose to keep β constant in order to make the whole exercise more transparent. In any case, our results did not change significantly under the alternative strategy.

consumption insurance is available, and only risk-free bonds can be used for consumption smoothing.

We first calculate the welfare differences between the complete-market and incomplete-market economies, for varying degrees of persistence in idiosyncratic shocks. This exercise measures the overall welfare cost of market incompleteness. We then isolate the welfare loss from capital rental market frictions by comparing the welfare of our second ($\lambda = 7.5$) and third ($\lambda = 1.5$) incomplete-market economies with that of our first incomplete-market economy ($\lambda = \infty$). Finally, we explore the forces behind the welfare cost of market incompleteness by analyzing the equilibrium allocations and prices of the incomplete-market economies, again for varying degrees of persistence in idiosyncratic shocks.

Our metric of welfare is the perfectly-egalitarian measure of economy-wide wellbeing. In particular, the welfare \mathcal{W} in a given economy is computed by integrating the value functions of individuals with different wealth and entrepreneurial productivity,

$$\mathcal{W} = \sum_{e \in \mathcal{E}} \mu(e) \int_0^{\infty} v(a, e) G_{\infty}(da|e).$$

The integration is with respect to the stationary distribution of wealth and entrepreneurial productivity, and the value function $v(a, e)$ is defined in (1).

The left panel of Figure 1 plots the overall welfare costs of market incompleteness in our model. We plot the welfare cost of our three incomplete-market economies ($\lambda = \infty$, 7.5, and 1.5) relative to the complete-market benchmark. The welfare costs are in units of permanent consumption compensation.⁵

The solid line is the welfare cost for the incomplete-market economy with perfect capital rental markets ($\lambda = \infty$), in the stationary equilibria corresponding to different degrees of persistence in idiosyncratic shocks. Here we are reproducing the well-known result that the welfare cost from missing consumption insurance can be substantial if the shocks are very persistent (Deaton, 1991; Huggett, 1993; Aiyagari, 1994). For the case of purely transitory shocks ($\rho = 0$), the welfare cost amounts to about ten per cent of permanent consumption, while it rises to about 25 percent for very persistent shocks ($\rho = 0.99$).⁶ This follows the standard intuition from Bewley models: It is harder to self-insure against more persistent shocks.

⁵We report the permanent consumption compensation that is required to make an individual indifferent ex ante between inhabiting the complete-market economy (CM) and a particular incomplete-market economy i : $1 - (\mathcal{W}^i / \mathcal{W}^{CM})^{1/(1-\gamma)}$.

⁶Even ten per cent in terms of permanent consumption is a rather large welfare cost in a standard Bewley model. Our calibration strategy of focusing on the right tail of (entrepreneurial) income distribution leads to an income inequality larger than estimates based on labor earnings data alone, and is subsequently responsible for this large welfare cost.

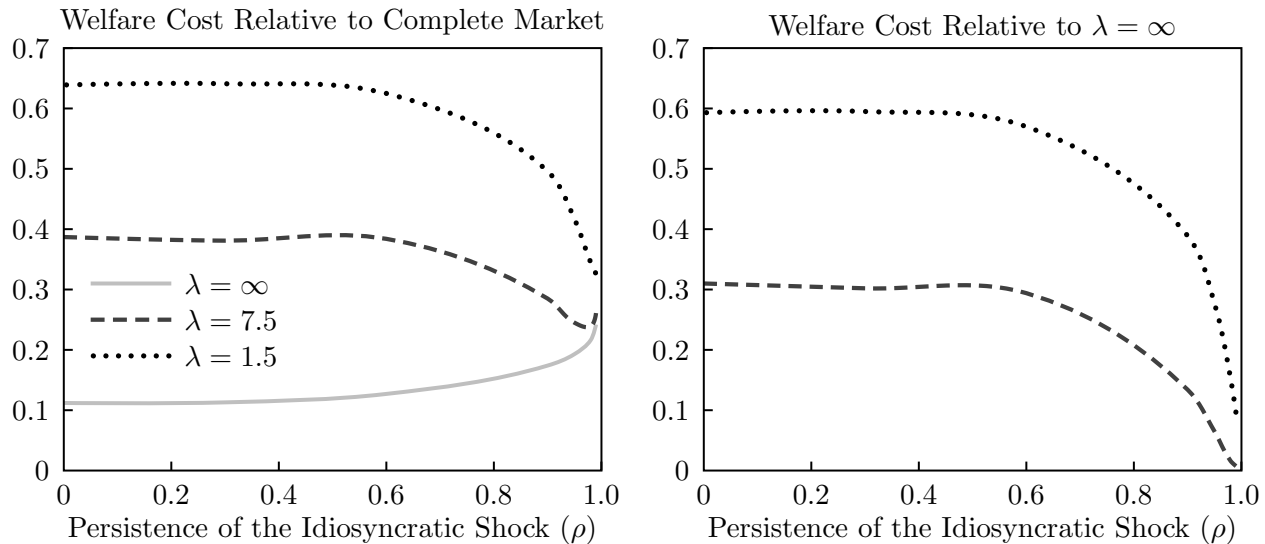


Fig. 1: Welfare Cost of Market Incompleteness. In the left panel, each line depicts the overall welfare cost of market incompleteness relative to the complete-market benchmark. In all three economies, a risk-free bond is the only instrument of consumption smoothing. They differ in the degree of frictions in the capital rental market ($\lambda = \infty$, 7.5, and 1.5). We report, in units of permanent consumption, the compensation required to make an individual indifferent ex ante between the complete-market economy and a particular incomplete-market economy. In the right panel, we plot the welfare costs from imperfect capital rental markets relative to the incomplete-market economy with perfect capital markets ($\lambda = \infty$). In both panels, welfare costs in the stationary equilibria are plotted against the corresponding shock persistence (ρ).

The dashed line ($\lambda = 7.5$) and the dotted line ($\lambda = 1.5$) in the left panel show the welfare costs of overall market incompleteness: the costs from capital rental market frictions as well as those from missing consumption insurance. The welfare loss from market incompleteness in our model can be substantially larger than that from a model that only focusses on missing consumption insurance. For instance, with a high degree of frictions in the capital rental market ($\lambda = 1.5$, dotted line), the overall welfare cost of market incompleteness is over 60 per cent of permanent consumption when the shock is purely transitory, and remains above 30 per cent when the shock is very persistent.

In the right panel of Figure 1 we plot the welfare difference between our incomplete-market economy with perfect capital markets ($\lambda = \infty$) and those with imperfect capital markets ($\lambda = 1.5$ and 7.5). This figure essentially isolates the welfare costs of imperfect capital rental markets from those of missing consumption insurance. Comparing the dotted and the dashed lines in the right panel with the solid line in the left panel, we can conclude that the adverse welfare impact of imperfect capital rental markets can be substantially larger than that of imperfect insurance, especially for low persistence of

shocks.⁷

Moreover, while the welfare cost from missing consumption insurance rises with the persistence of shocks, that from imperfect capital rental markets is monotonically decreasing in the persistence of entrepreneurial productivity shocks. In our model with imperfect capital rental markets, productive entrepreneurs need to accumulate wealth so that they can overcome the collateral constraint and self-finance their profitable projects. Self-financing only works when shocks are persistent enough and entrepreneurs are given the incentive and time to accumulate wealth. If shocks are transitory, entrepreneurs who come across a profitable project do not expect this opportunity to last long, and hence do not have a strong incentive to patiently accumulate wealth to overcome the collateral constraint. Consequently, they will remain undercapitalized. As an aside, our result on the persistence of shocks and the impact of capital market frictions has a broader implication: When individuals in the model are not given the chance to overcome capital market frictions through self-financing—perhaps because they are assumed to work for only one period or to have an exogenously-imposed saving behavior, the impact of capital market frictions can be exaggerated.

We draw the conclusion that, while persistent shocks undermine individuals' ability to self-insure, they enhance the viability of self-financing. This intuition predicts that the welfare effect of the persistence of shocks is potentially non-monotone. Indeed, for intermediate degrees of frictions in the capital rental market ($\lambda = 7.5$, dashed line), the welfare cost is U-shaped when plotted against the persistence of idiosyncratic shocks. The right (and upward) arm of the U reflects the difficulty of self-insurance against very persistent shocks; The left (and downward) arm, the difficulty of overcoming capital market frictions through self-financing when individuals' entrepreneurial opportunities are short-lived.

To better understand the welfare effect of the persistence of shocks, we look at the equilibrium allocations and prices. In Figure 2 we report how the average and the dispersion of individual consumption in the stationary equilibria, along with wage and interest rates, respond to changes in the persistence of shocks, for each of our three incomplete-market economies. The mean consumption (top left panel) is relative to its value in the complete-market benchmark, while the dispersion measure of consumption is its coefficient of variation (top right panel).

From the top panels of Figure 2, we discern a clear trade-off between the mean and the dispersion of consumption in the stationary equilibria indexed by the persistence of shocks.

⁷Note that the two panels have the same scale, facilitating visual comparison.

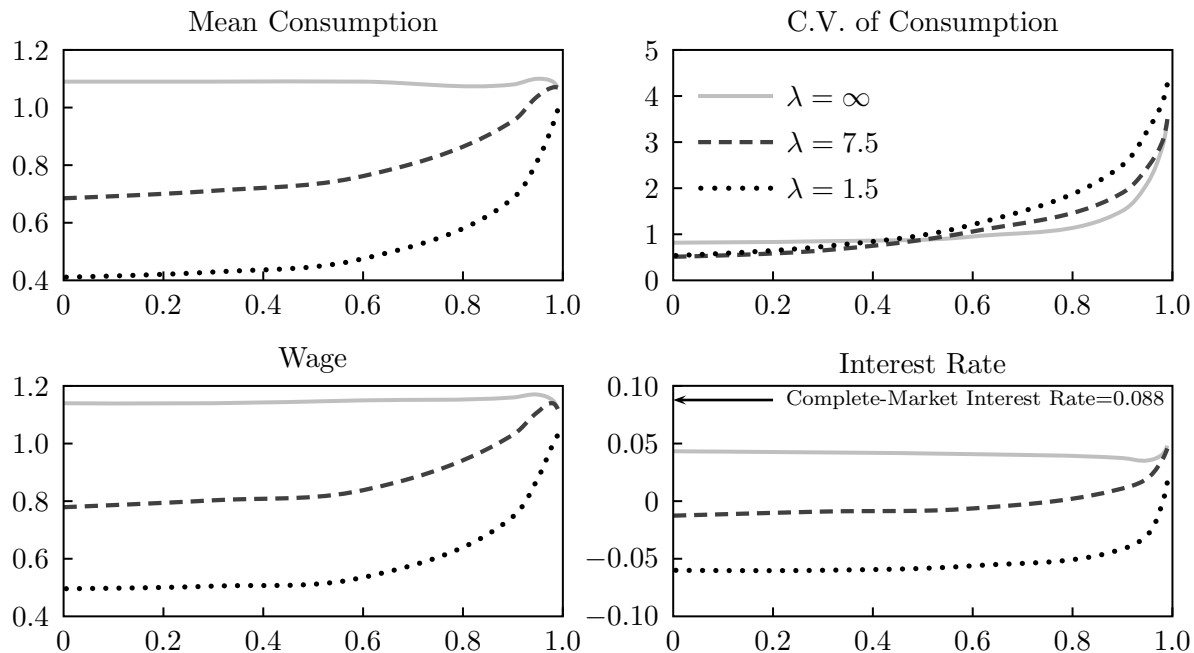


Fig. 2: Allocations and Prices in the Stationary Equilibria. We plot the mean and the coefficient of variation of consumption, along with wage and interest rates, in the stationary equilibria of our three incomplete-market economies ($\lambda = \infty$, 7.5, and 1.5). The mean consumption and the wage rate are relative to their respective values in the complete-market benchmark. In all four panels, the quantities and prices in the stationary equilibria are plotted against the corresponding shock persistence (ρ).

Frictions in the capital rental market can substantially reduce the average consumption. This effect is largest when shocks are transitory: Self-financing can not make up for the lack of external financing. On the other hand, with persistent shocks, self-insurance is not an effective way of smoothing consumption, and consumption inequality worsens in the equilibrium. As the left panel of Figure 1 shows, with a high degree of capital market frictions ($\lambda = 1.5$, dotted line), the effect of shock persistence on the mean consumption dominates its effect on consumption dispersion, and the overall welfare cost is decreasing in shock persistence. On the contrary, with a well-functioning capital rental market ($\lambda = \infty$, solid line), shock persistence has only negligible effects on the mean consumption: The welfare cost is increasing in shock persistence. With an intermediate degree of capital market frictions ($\lambda = 7.5$, dashed line), there is a more interesting mean-variance trade-off, and the overall welfare cost is non-monotone in shock persistence.

In the bottom panels of Figure 2, we report the equilibrium wage and interest rates. Wage rates are relative to their level in the complete-market benchmark. The low wage in the economies with imperfect capital rental markets reflects the misallocation of capital across entrepreneurs and the resulting lower productive efficiency. In our incomplete-

market economy with perfect capital rental markets, the wage is higher than that in the complete-market benchmark (solid line, bottom left panel). This follows from the celebrated capital over-accumulation result of Aiyagari (1994). The wage in the economies with capital market imperfections rises with shock persistence. This pattern not only looks very similar to that of the mean consumption, but is also explained in the same way.

The bottom right panel shows the behavior of the interest rate. The interest rate with complete markets is equal to the subjective discount rate, regardless of the persistence of shocks (0.088). It is well-understood that, with incomplete markets, the equilibrium interest rate is lower than its level with complete markets (Bewley, 1980; Huggett, 1993; Aiyagari, 1994). In the economies with imperfect capital rental markets, the equilibrium interest rate tends to be even lower. The interest rate is driven lower by entrepreneurs' restricted demand for capital due to collateral constraints, and also by their higher saving rates due to the need for self-financing (a bigger supply of capital, all else equal). With persistent shocks, entrepreneurs can eventually overcome the collateral constraints, pushing up the equilibrium interest rate.⁸

In summary, two new results on the welfare cost of market incompleteness are obtained from our analysis. First, the welfare cost of imperfect capital rental markets can be substantially higher than that of imperfect consumption insurance. Second, this welfare cost of imperfect capital markets is particularly high when shocks are not persistent, because self-financing is not an effective substitute for external financing in those cases.

4 Concluding Remarks

Bewley models are a workhorse of modern macroeconomics. They have proven useful for understanding income and wealth inequality in the data, and for assessing the welfare consequences of market incompleteness. Numerous papers relied on these models to evaluate the macroeconomic and distributional effects of various economic policies, ranging from income taxation to pensions and health insurance.

One recurring criticism of Bewley models—starting perhaps with Stiglitz (1981)—has been that the source of market incompleteness is not explicitly modeled. Our paper partly addresses this issue by being explicit about the fundamental enforceability problem that gives rise to the collateral constraints in the capital rental market. Admittedly, we have fallen short of taking stance on which deep frictions are responsible for the lack of explicit

⁸See Buera and Shin (2008) for more discussions on the equilibrium interest rate. The lower bound on the interest rate here is $-\delta$. If we replace our no-borrowing constraint ($a \geq 0$) with the “natural” borrowing limit ($a \geq -w/r$), the equilibrium interest rate will be bounded from below, away from zero.

consumption insurance.

In this paper, we raise two additional issues with the conventional Bewley models. One is the asymmetric assumptions of market incompleteness on the consumption side and the production side. We explore the idea that the same enforcement or information problems that preclude consumption insurance may as well disrupt the functioning of the capital rental market. The other pertains to the way individual income processes are modeled. Instead of specifying the income process directly, we go one level deeper and consider how entrepreneurial productivity or business opportunities translate into income realizations.

Our paper demonstrates that some standard model implications might be overturned once we enrich Bewley models to address such additional issues. We envision many other useful ways of laying the foundations of Bewley models, and leave them for future research.

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