Ramsey Meets Bewley: Optimal Government Financing with Incomplete Markets

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Abstract

This paper studies optimal fiscal policy in an economy with heterogeneous households and incomplete markets. Relative to a representative-agent version of the model, the Ramsey planner takes into account the idiosyncratic income risk faced by heterogeneous households in a way that alters the model's prediction about the level of government debt. The simpler model with a representative agent has the government *accumulate* assets to minimize tax distortion in the long run. In contrast, with heterogeneous agents who face undiversifiable idiosyncratic risk that is sufficiently large relative to aggregate risk, the Ramsey planner chooses to *issue* debt and facilitate the precautionary saving of the private sector, even at the cost of extra tax distortion. I interpret these outcomes in terms of the strengths of two competing insurance motives that concern the Ramsey planner: aggregate tax smoothing and individual consumption smoothing.

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This paper studies how a government should pay for its expenditure when risk cannot be shared perfectly—neither between the government and households, nor among households. This market incompleteness motivates precautionary saving on the part of all economic agents. The precautionary saving of households is a well-understood concept. Although not as well-known, a government also has a precautionary saving motive when markets are incomplete. On the subject of ancient governments, David Hume wrote the following commentary in 1752.

> It appears to have been the common practice of antiquity, to make provision, during peace, for the necessities of war, and to hoard up treasures before-hand [...]; without trusting to extraordinary impositions, much less to borrowing, in times of disorder and confusion. [David Hume, "Of Public Credit."]

This is exactly the optimal Ramsey plan that Aiyagari, Marcet, Sargent, and Seppälä (2002) prescribe in their incomplete-market representative-agent economy. In their analysis, the debt repayment from the households to the government is a lump-sum transfer, unlike the revenue from distortionary income taxes. If the government partly defrays expenses out of its asset holdings, the tax rate and associated deadweight loss can both be lowered. Thus a benevolent government will build up a war chest of assets as a precaution against adverse expenditure shocks and smooth taxes over time.

If the government is accumulating assets in a closed economy, the households must be borrowing from the government. How can this observation be reconciled with the ordinary precautionary saving by households? The answer is that the optimal fiscal policy balances the precautionary saving of the representative household against the government's desire to reallocate tax distortion over time and across states. A high level of household debt (government asset) helps the government reduce taxes in the future. Lower taxes, in turn, boost production and consumption. Thus, for the representative household, borrowing does not come at the expense of less consumption in the future. This is how the precautionary saving motives of the household and the government are effectively bundled together. It should also be noted that the existence of a representative household presupposes perfect risk sharing among households.

In this paper, I set up an environment where the two precautionary saving motives become unbundled. For this purpose, I introduce idiosyncratic income risk for households, in addition to the aggregate government expenditure shock. The government is a benevolent Ramsey planner, and the only fiscal instruments available to the government are proportional labor income tax and risk-free one-period bond. The idiosyncratic risk cannot be diversified away, but households can issue and purchase risk-free bonds for self-insurance. The presence of idiosyncratic risk not insured by the government brings out an independent role for the precautionary saving motive of the households.

Because asset market clearing dictates that the net supply of risk-free bonds be zero in a closed economy, the precautionary saving motives of the households and of the government are pitted against each other. If the agents' idiosyncratic income risk is large enough relative to the government expenditure risk, the Ramsey planner finds it optimal to issue debt to facilitate the self-insurance of the private sector, even at the cost of extra tax distortion in the long run.

Even with a representative consumer, Chari, Christiano, and Kehoe (1995, p.366) describe the Ramsey problem with incomplete markets as a 'difficult exercise' because of the proliferation of implementability constraints. It is therefore not surprising that there are few papers that analyze such problems.

Barro (1979) exploits an analogy to a permanent-income model of consumption (Hall, 1978) and formalizes the idea of intertemporal tax-smoothing. His main result is that tax rates should depend only on permanent government spending and on the level of government debt. As is well-known, it implies a random walk in taxes. As for the government debt policy, the government should issue debt only when the current spending is above the permanent level. Otherwise, it should accumulate assets.

Aiyagari et al. (2002) pursue this issue further by recasting Barro's model in a general equilibrium framework with incomplete markets. In their resulting equilibrium, optimal tax rates exhibit a near unit-root component, an affirmation of Barro's random walk. As for the level of government debt, the Ramsey plan dictates that the government should accumulate assets to minimize tax distortion in the long run. In one particular case, they prove that the government will accumulate enough assets that it can finance its entire expenditure with interest income and set all future tax rates to zero.

In reality, most governments issue debt. It seems even more unlikely that all taxes will vanish in the long run. I point to the representative-agent assumption as the smoking gun behind these counterfactual implications. In an economy with heterogeneous households who face idiosyncratic income risk that is sufficiently large relative to government expenditure risk, it is optimal for the government to issue debt. With a positive quantity of government debt outstanding, tax rates are bounded away from zero, and their limiting distribution is non-degenerate.

Admittedly, that public debt helps individual consumption smoothing in the presence of undiversifiable idiosyncratic risk is not a new idea. At the opposite extreme of Aiyagari et al. (2002) lie Aiyagari and McGrattan (1998), who discuss the role of public debt as private liquidity. They assume that government bonds are equally distributed among households, who use their bond holdings as collateral. A higher level of government debt therefore relaxes the households' borrowing constraint, and in turn alleviates the problem of capital over-accumulation (Aiyagari, 1994). At the same time, more debt comes with more tax distortion. Aiyagari and McGrattan compare welfare across different steady states, and pick the best debt-tax combination. In their model, there is no aggregate risk, and thus there is no active tax smoothing motive on the part of the government—in fact, a government is no more than a sequence of budget constraints.

In a unified analysis, I explicitly account for the dual functions of government debt—tax smoothing and individual consumption smoothing—and study how the relative importance of aggregate versus idiosyncratic risk determines the optimal financing decision of the government. It is shown that microeconomic heterogeneity (undiversifiable individual income risk) influence macroeconomic fiscal variables (government debt and taxes) in a non-trivial manner.

1 The Economy

In this section, I propose the simplest possible model with all the necessary ingredients: distortionary taxation, incomplete markets, aggregate and idiosyncratic risk. Risk and tax distortion create the need for insurance. At the same time, I limit the available means of insurance to elicit and amplify the precautionary saving motive of economic agents.

1.1 Households

There are two types of households in the economy. They behave competitively, and they are identical ex ante. Within a type, all households remain identical ex post. However, a type-1 household and a type-2 household undergo different realizations of idiosyncratic shocks. The idiosyncratic risk takes the form of efficiency unit of labor, θ_t^i, where i indexes household types, and t denotes periods. Each type is assumed to be of measure one.

Households order their consumption-leisure allocation streams by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^i, x_t^i),$$

where c and x respectively stand for consumption and leisure. Throughout the paper, u is specified to be additively separable in the two arguments. u is assumed to be strictly concave and increasing in both arguments. Furthermore, u is twice continuously differentiable, and $\lim_{c\downarrow 0} u_c = +\infty$ and $\lim_{x\downarrow 0} u_x = +\infty$.

Both types of households are endowed with ξ units of perfectly divisible time each period, which can be either consumed as leisure or devoted to production.

1.2 Technology

Labor is the only input of production, and the production technology exhibits constant returns to scale: $y_t^i = \theta_t^i(\xi - x_t^i)$, where θ_t^i is the efficiency unit of labor for type-i households. I assume perfectly competitive markets for goods and labor, and hence the profit-maximizing firms will not receive much attention in the analysis to follow. θ_t^i doubles as each household's market wage.

1.3 Government

The government in this economy is benevolent—that is, the government maximizes the welfare of the whole population. The welfare criterion of the government is an equally-weighted sum of the utility of the households:

$$\mathbb{E}_0 \sum_{i=1}^{2} \sum_{t=0}^{\infty} \beta^t u(c_t^i(s^t), x_t^i(s^t)).$$

The government is a Ramsey planner. In the initial period, it announces a system of prices and taxes over all possible history realizations. I assume there is a commitment technology, or an institution, available to the government.

The government expenditure, g_t, follows an exogenous stochastic process. I assume that $\{g_t\}_{t=0}^{\infty}$ does not directly enter the utility of the households.[1] The exogenous nature of the government expenditure may not seem innocuous. However, there is nothing unique in the way the government expenditure shock affects the economy. The whole analysis goes through even if the government expenditure shock is replaced by, for example, an aggregate productivity shock.

There is a flat-rate tax on labor income, and it is the only source of tax revenue. The government finances its expenditure with the income tax revenue and by borrowing or saving. The model rules out any lump-sum transfer between the households and the government. This tax distortion, together with the expenditure shock, gives the government the need for tax smoothing.

1.4 Idiosyncratic and Aggregate Risk

The three-tuple of household efficiency units of labor and government expenditure, $s_t = (\theta_t^1, \theta_t^2, g_t)$, is drawn from a compact set $\mathcal{S}$, and all the realizations are publicly observable. The stochastic process $\{s_t\}_{t=0}^{\infty}$ is defined on the infinite Cartesian product of $\mathcal{S}$ with the

[1] Everything remains the same if I alternatively assume that the utility derived from g_t is strongly separable from $u(c_t, x_t)$.

σ-algebra induced by the cylinder sets and probability measure π. For any history $s^t = (s_0, \ldots, s_t)$, I use $\pi(\mathcal{Z})$ to denote the probability that $s^t \in \mathcal{Z}$, where $\mathcal{Z} \subset \mathcal{S}^{t+1}$ is any measurable set.

The probability distribution is restricted to be symmetric across the households ex ante, and it is further assumed that θ_t^1 and θ_t^2 are perfectly negatively correlated with each other. More importantly, (θ_t^1, θ_t^2) and g_t are mutually independent. These three conditions ensure that the idiosyncratic risk is literally idiosyncratic. By drawing a clear line between these two categories of risk, I can make exact statements about the relative importance of aggregate versus idiosyncratic risk in the following analysis.

1.5 Asset Markets

To study how market incompleteness influences a government's financing decisions, I have to restrict the means of risk sharing among all economic agents in the economy. In particular, I allow only one type of security to be traded: risk-free one-period discount bonds, whose par value is one unit of consumption good. The price of the bond is the reciprocal of the gross real interest rate. The government and households can freely issue and purchase bonds subject to certain debt limits, which are defined below. IOUs issued by households are perfectly interchangeable with government bonds. If the asset market is to clear in this closed economy, the net saving by the households must be equal to the net borrowing by the government, and vice versa.

Assumptions made about the debt limits of the households and the government will affect the distribution of asset holdings in the economy. If, for example, the households are not allowed to borrow at all, then the government will issue debt with probability one. To minimize the impact of such assumptions on the equilibrium level of private and public debt, ideally, one would impose on both the government and the households the most lenient borrowing constraint—a "natural" debt limit. A natural debt limit only stipulates that any outstanding liability be repaid almost surely. However, the computation of natural debt limits is not simple in this model. The calculation of natural debt limits at any given period presupposes a complete characterization of the optimal tax and interest rates for all future periods. Thus, in practice, I approximate the natural debt limits of the households and the government by $\underline{A}$ and $\bar{B}$, respectively: $\underline{A} = \frac{\beta}{1-\beta}\left(\theta_{min}\xi - \frac{g_{max}}{2}\right)$ and $\bar{B} = \frac{\beta}{1-\beta}\left(\min\{\theta_t^1\xi + \theta_t^2\xi\} - g_{max}\right)$, where θ_{min} is the lowest possible realization of θ_t^i, and g_{max} is the highest possible realization of g_t. This is akin to how Aiyagari (1994) constructs natural debt limits from the "worst" sample path.

2 Incomplete Market Ramsey Problem

First, I define a competitive equilibrium, and then pose the associated Ramsey problem.

Definition 1 *A competitive equilibrium in an incomplete market economy consists of sequences of bond prices, $\{q_t(s^t)\}_{t=0}^{\infty}$, tax rates, $\{\tau_t(s^t)\}_{t=0}^{\infty}$, allocations, $\{c_t^i(s^t), x_t^i(s^t)\}_{t=0}^{\infty}$ for $i = 1, 2$, household asset holdings, $\{a_{t+1}^1(s^t), a_{t+1}^2(s^t)\}_{t=0}^{\infty}$, and government debt, $\{b_{t+1}(s^t)\}_{t=0}^{\infty}$, that satisfy the following.*

1. *Given $\{q_t(s^t), \tau_t(s^t)\}_{t=0}^{\infty}$, $\{c_t^i(s^t), x_t^i(s^t)\}_{t=0}^{\infty}$ maximizes the household objective function subject to the budget constraint and the debt limit for each s^t:*

$$c_t^i(s^t) + q_t(s^t)a_{t+1}^i(s^t) \le (1 - \tau_t(s^t))\theta_t^i(s^t)(\xi - x_t^i(s^t)) + a_t^i(s^{t-1}), \quad a_{t+1}^i(s^t) \ge -\underline{A}.$$

2. *The government budget constraint is satisfied and the debt limit of the government is not violated for each s^t:*

$$g_t(s^t) + b_t(s^{t-1}) \le \tau_t(s^t)\sum_{i=1}^{2}\theta_t^i(s^t)(\xi - x_t^i(s^t)) + q_t(s^t)b_{t+1}(s^t), \quad b_{t+1}(s^t) \le \bar{B}.$$

3. *The goods market clears for each s^t: $c_t^1(s^t) + c_t^2(s^t) + g_t(s^t) = \sum_{i=1}^{2}\theta_t^i(s^t)(\xi - x_t^i(s^t))$.*
4. *The asset market clears for each s^t: $a_{t+1}^1(s^t) + a_{t+1}^2(s^t) = b_{t+1}(s^t)$.*

The corresponding Ramsey problem is to solve for $\{q_t(s^t), \tau_t(s^t)\}_{t=0}^{\infty}$ that implements the incomplete market competitive equilibrium corresponding to the maximal value of the government objective function.

More formally, the Ramsey problem can be written down as follows. For notational convenience, the dependence of variables on history (s^t) is suppressed.

$$\max \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi(s^t)\left\{u(c_t^1, x_t^1) + u(c_t^2, x_t^2)\right\}$$

$$s.t. \quad c_t^1 + c_t^2 + g_t \le \theta_t^1(\xi - x_t^1) + \theta_t^2(\xi - x_t^2) \tag{1}$$

$$c_t^i + q_t a_{t+1}^i \le (1 - \tau_t)\theta_t^i(\xi - x_t^i) + a_t^i, \ \forall i \tag{2}$$

$$g_t + a_t^1 + a_t^2 \le \tau_t\{\theta_t^1(\xi - x_t^1) + \theta_t^2(\xi - x_t^2)\} + q_t\{a_{t+1}^1 + a_{t+1}^2\} \tag{3}$$

$$\frac{u_{x,t}^1}{\theta_t^1 u_{c,t}^1} = 1 - \tau_t = \frac{u_{x,t}^2}{\theta_t^2 u_{c,t}^2} \tag{4}$$

$$\begin{cases} q_t = \beta\frac{\mathbb{E}_t u_{c,t+1}^i}{u_{c,t}^i}, \ \forall i, \text{ if } a_{t+1}^i > -\underline{A} \text{ for all } i \\ q_t = \max_i\left\{\beta\frac{\mathbb{E}_t u_{c,t+1}^i}{u_{c,t}^i}\right\}, \text{ otherwise} \end{cases} \tag{5}$$

$$a_{t+1}^i \ge -\underline{A}, \ \forall i, \text{ and } a_{t+1}^1 + a_{t+1}^2 \le \bar{B} \tag{6}$$

The maximization is over $\{c_t^i(s^t), x_t^i(s^t), a_{t+1}^i(s^t), \tau_t(s^t), q_t(s^t)\}$ for all possible s^t and for $i = 1, 2$. Constraints (1) through (6) must hold for all s^t. By the asset market clearing condition, b_t is replaced with $a_t^1 + a_t^2$. (1) is the resource constraint, and (2) is the budget constraint of the households. The budget constraint of the government, (3), automatically follows from (1) and (2), and can be omitted. Implementability constraints, (4) and (5), ensure that the households optimize given the taxes and interest rates. The within-period labor-leisure decision is encapsulated in (4), while the intertemporal consumption-saving decision is in (5). If the debt limit is binding for either type of households, the risk-free bond is priced by the type with the higher intertemporal marginal rate of substitution. The debt limits in (6) are as defined in Section 1.5.

This infinite sequence problem can be transformed into a tractable recursive system, as in Kydland and Prescott (1980). The corresponding Bellman equation is in Appendix C.

3 A Two-Period Economy

Before proceeding to the numerical analysis of the full-blown model, I present a two-period incomplete market Ramsey problem that conveys the thrust of the model, and highlight the driving forces behind the results.

The two types of households order their consumption-leisure allocations by

$$\mathbb{E}_0 \left[\log(c_0^i) + \log(x_0^i) + \log(c_1^i) + \log(x_1^i)\right]. \tag{7}$$

The government expenditure at $t = 0$ (g_0) is zero, but g_1 is either $g_L = -\Delta$ or $g_H = \Delta$ with equal probability, where $0 \leq \Delta < 1$. $g_L < 0$ means that, instead of consuming resources, the government brings consumption goods into the economy, allowing the private sector to consume more than it produces. In this case, the government is assumed to subsidize the households with negative income taxes. Both types of households have $\theta_0^i = 1$. Independently of the government expenditure shock, there is undiversifiable idiosyncratic risk to the households at $t = 1$. With equal probability, the two-tuple of their efficiency units of labor, (θ_1^1, θ_1^2), is either $(1-\sigma, 1+\sigma)$ or $(1+\sigma, 1-\sigma)$, where $0 \leq \sigma < 1$. Note that σ and Δ are the standard deviations of the idiosyncratic and aggregate shocks, respectively.

Production technology and available fiscal instruments are as described in Section 1, and ξ is normalized to one. The only security traded in the economy is risk-free one-period bonds. The households, as well as the government, can issue and purchase the bond in any quantity, but they have to deliver on their promises. The households and the government enter the initial period with zero asset holdings. This Ramsey problem is explicitly written down in Appendix A. Ex ante symmetry across the households implies $a_1^1 = a_1^2$. Then, the asset market clearing implies that $b_1 = 2a_1$, where $a_1 = a_1^1 = a_1^2$.

The Ramsey problem does not have a closed-form solution, and the non-linear implementability constraints preclude general characterizations of the outcome. However, given the assumption of log utility and the normalization of the aggregate shock ($g_0 = \mathbb{E}_0 g_1 = 0$), a few local results can be established around $b_1 = a_1 = 0$, where there is no trading in the asset market. I refer to the outcome with $b_1 = a_1 = 0$ as the balanced-budget equilibrium.

Proposition 1 *When there is more idiosyncratic risk than aggregate risk ($\sigma > \Delta$), the government can improve welfare, compared to the balanced-budget equilibrium, by issuing debt. When $\sigma < \Delta$, the government can improve welfare by accumulating IOUs issued by the households. Finally, when $\sigma = \Delta$, the balanced-budget equilibrium is a local maximum.*

Proof See Appendix B.

Two special cases are of particular interest.

Corollary 1 (Representative agent) *When $\sigma = 0$ and $\Delta > 0$, compared to the balanced-budget equilibrium, the government can improve welfare by accumulating IOUs issued by the households.*

Corollary 2 (No aggregate shock) *When $\sigma > 0$ and $\Delta = 0$, compared to the balanced-budget equilibrium, the government can improve welfare by issuing debt.*

Further numerical analysis confirms that the above local results hold globally as well. The optimal quantity of government debt issued at $t = 0$ is plotted in Figure 1 against σ, the standard deviation of the idiosyncratic shock. Each curve corresponds to different levels of government expenditure variability, Δ.

The debt repayment by the private sector is a lump-sum transfer, unlike the distortionary income taxes. If the government partly defrays expenses out of its asset holdings, the tax rate is lower than when the whole expenses are funded only with tax revenue. So is the deadweight loss. To minimize the overall tax distortion given the distributional assumptions on g_0 and g_1, the government will smooth taxes by generating surplus in the initial period and purchasing bonds issued by the households.[2] Such tax smoothing implies a shift in consumption from $t = 0$ to $t = 1$. As a result, the equilibrium interest rate must be higher than in the balanced-budget equilibrium. With a higher interest rate, it becomes less costly, in terms of today's distortion, to reduce that of tomorrow.

[2] If the government's objective function were to be quadratic, the balanced-budget equilibrium would be optimal given the symmetric distribution of g_1 around g_0.

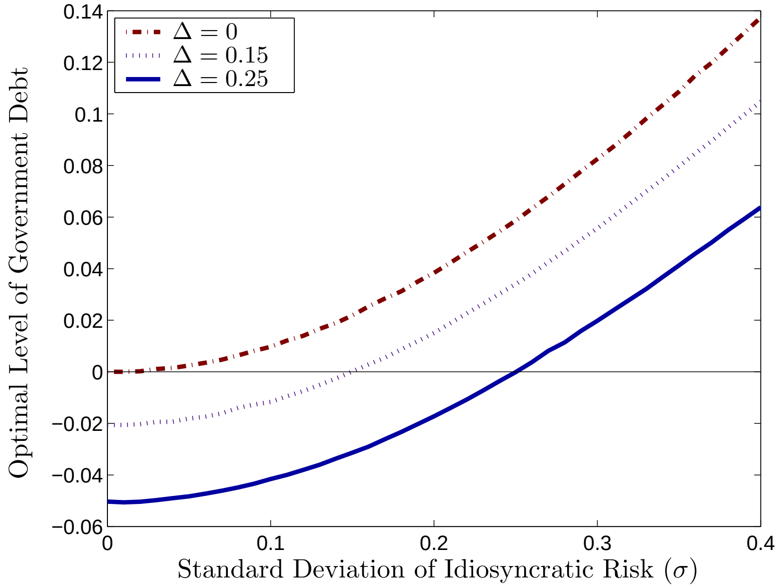

Fig. 1: The optimal level of public debt issued at $t = 0$. The standard deviation of the undiversifiable idiosyncratic shock, σ, is on the horizontal axis. For each curve, $g_0 = 0$, while g_1 could be either $g_L = -\Delta$ or $g_H = \Delta$. A negative level of debt means that the government is accumulating IOUs issued by the households.

On the other hand, the households need a buffer stock of assets for self-insurance against the idiosyncratic risk. Given that there is zero net supply of assets in this closed economy, a race between the two precautionary saving motives is inevitable.

Figure 1 captures the idea of such competing insurance motives. In the region where the aggregate shock is more important ($\sigma < \Delta$), the government takes over the role of insuring the economy against the expenditure shock by accumulating IOUs issued by the households. If the idiosyncratic risk weighs in more heavily ($\sigma > \Delta$), the precautionary saving motive of the households prevails, driving the government to issue bonds and accommodate their self-insurance needs. When the standard deviation of government expenditure increases from 0.15 to 0.25, the government's precautionary saving against the aggregate risk intensifies, as evidenced by the downward shift of the curve in Figure 1. The government borrows less, or saves more.[3]

Note that $\sigma = 0$, as in Corollary 1, corresponds to the representative-agent case, where the households remain identical ex post. As Figure 1 shows, the government accumulates private IOUs issued by the representative consumer. This is what Aiyagari et al. (2002)

[3]This outcome contrasts with the findings of Holmström and Tirole (1998). In their model with risk-neutral firms and no tax distortion, there is no role for government bonds when all the shocks are idiosyncratic. In the presence of aggregate risk, it is optimal for the government to issue bonds and provide liquidity.

find in their incomplete-market representative-agent models. Given that the benevolent government smooths out the consumption of the households in the face of the aggregate risk by accumulating assets, the precautionary saving motive of the households is subsumed into the optimal fiscal policy.

On the other hand, the top curve in Figure 1 represents the world of Aiyagari and McGrattan (1998), Bewley (1980, 1983) and Huggett (1993), where there is no aggregate risk. In the absence of the government's precautionary saving motive, it is always optimal to have a positive net supply of "outside money" or government debt in the economy and help the households insure themselves against undiversifiable idiosyncratic risk.

The equilibrium consumption allocations offer an alternative way of looking at the race between the two precautionary saving motives. An inevitable consequence of issuing public debt is that the government now has to generate extra revenue to repay the households at $t = 1$. The incremental taxation incurs additional distortion, lowering the aggregate output and consumption. In Appendix A, I show that the gross output at $t = 1$ is given by $y_1 = \frac{1+g_1+a_1+\sqrt{(1-g_1-a_1)^2-4a_1}}{2}$. For fixed g_1 and σ, total output decreases in response to an increase in government debt ($2a_1$). Of course, so does the difference between the consumption of the more productive households and that of their less productive counterparts, measured by $\frac{c_{1h}}{c_{1l}} = \frac{2+\sigma-y_1}{2-\sigma-y_1}$. This trade-off between insurance against idiosyncratic risk and tax distortion lies behind the race.

4 Optimal Fiscal Policy: Numerical Examples

This section revisits the original problem laid out in Section 2. To better demonstrate how the optimal fiscal policy responds to aggregate and idiosyncratic risk, I create three example economies and collate the results. These three economies share the same preference, technology, asset market structure and fiscal instruments, but are subject to exogenous shocks with different magnitude and persistence.

I define the utility function $u : \mathbb{R}_+ \times (0, \xi] \to \mathbb{R}$ as $u(c, x) = \log c + \log x$, and let $\beta = 0.95$. For computational tractability, I assume that the set $\mathcal{S}$ has only four elements: $\mathcal{S} = \{(\theta_l, \theta_h, g_L), (\theta_h, \theta_l, g_L), (\theta_l, \theta_h, g_H), (\theta_h, \theta_l, g_H)\}$, where $\theta_l < \theta_h$ and $g_L \leq g_H$. I assume that the stochastic process $\{s_t\}_{t=0}^{\infty}$ is governed by a four-state Markov chain. For stability in numerical implementation, I normalize ξ to three. Thus, for all i and t, $x_t^i \in (0, 3]$, and a household's output is now $y_t^i = \theta_t^i(3 - x_t^i)$.

In all three economies, the government expenditure shock is assumed to be independently and identically distributed, with g_t being equally likely to be g_L or g_H. Let Π_θ be the two-by-two transition matrix for the idiosyncratic risk. The first row and column of Π_θ correspond to $(\theta_t^1, \theta_t^2) = (\theta_l, \theta_h)$, and the second to $(\theta_t^1, \theta_t^2) = (\theta_h, \theta_l)$. In accordance with the restrictions laid out in Section 1.4, Π_θ is a symmetric matrix with ϕ's on the diagonal and $(1-\phi)$'s off the diagonal. ϕ parameterizes the persistence of the idiosyncratic risk. It is also assumed that the idiosyncratic and the aggregate shocks are mutually independent.

Table 1 summarizes how the three economies differ from one another. In the table, μ_θ, σ_θ and ρ_θ stand for the mean, standard deviation and auto-correlation of order one for the idiosyncratic shock. The corresponding moments of the aggregate shock are marked with subscript g.

	Idiosyncratic Risk	Aggregate Risk
Economy 1	$\mu_\theta = 1$, $\sigma_\theta = 0.3$, $\rho_\theta = 0$	$\mu_g = 1$, $\sigma_g = 0$
Economy 2	$\mu_\theta = 1$, $\sigma_\theta = 0.3$, $\rho_\theta = 0$	$\mu_g = 1$, $\sigma_g = 0.15$, $\rho_g = 0$
Economy 3	$\mu_\theta = 1$, $\sigma_\theta = 0.3$, $\rho_\theta = 0.5$	$\mu_g = 1$, $\sigma_g = 0.15$, $\rho_g = 0$

Table 1: Exogenous shock processes for the example economies

In the first economy, there is no aggregate shock ($g_L = g_H$), and the individual shock is not serially correlated. I add in stochastic government expenditure to create the second economy. By comparing the first two economies, one can determine how the relative magnitude of the aggregate versus idiosyncratic shocks affects the optimal financing decision of the government. Finally, the third economy is designed to assess how the persistence of shocks affects the optimal fiscal policy, a question that cannot be answered with the two-period model. In Appendices C and D, I transform the Ramsey problem into a dynamic programming problem, and then discuss how the Bellman equation is solved in practice.

4.1 Government Debt and Tax Rates

I summarize the properties of the optimal fiscal policy by tabulating the simulated moments of relevant variables. All the sample means, standard deviations (SD), and auto-correlation coefficients of order one (AR1) are constructed from 50 sample paths of length 500 each. In all the simulations, the initial asset holdings of the households and the government are zero.

In the first economy, where there is no aggregate shock, the mean level of the government debt is 2.825 or about 94% of the average output of the economy. The mean tax rate on labor income is 39%. Taxes are higher than those in a balanced-budget regime (34% on average, not reported in Table 2) because the government has to pay for the interests on its outstanding debt in addition to the exogenous spending. However, the issuance of government debt facilitates the consumption smoothing of the households, and more than offsets the welfare loss from the extra tax distortion.

	Government Debt			Tax Rate		
	Mean	SD	AR1	Mean	SD	AR1
Economy 1	2.825	0.870	0.988	0.390	0.028	0.404
Economy 2	2.295	1.067	0.970	0.386	0.035	0.642
Economy 3	2.469	1.189	0.981	0.395	0.037	0.671

Table 2: Simulated moments of government debt and taxes

As aggregate risk is introduced (Economy 2), the mean level of government debt decreases to 2.295 or about 77% of the average output. It indicates that the underlying mechanism of Section 3—the downward shift of the curve in Figure 1—is not limited to two-period models. The government's precautionary saving motive against the aggregate risk partially offsets the precautionary saving of the households. Both government debt and taxes are more volatile than those in Economy 1, which is natural given the addition of aggregate risk.

In Economy 3, with more persistent idiosyncratic risk, the precautionary saving motive of the households becomes stronger, and there is more demand for risk-free assets. As a consequence, the mean level of government debt in Economy 3 exceeds that in Economy 2, again confirming the idea that the relative strengths of the precautionary saving motives determine the optimal level of government debt.

The serial correlation of tax rates is one of the moments that the optimal taxation literature has focused on. Barro (1979) postulates that tax rates, for any given government expenditure shock process, follow a random walk. Lucas and Stokey (1983) conclude that, with complete markets, tax rates should mirror the serial correlation of the underlying government expenditure shock, subverting Barro's intuition. Finally, Aiyagari et al. (2002) identify a near unit-root component in the optimal tax rates, affirming Barro's result.

Table 2 shows that the tax rates of Economies 2 and 3 are much more persistent than the i.i.d. government expenditure shock. In the example economies, tax rates are determined by not only the government expenditure, but also the level of government debt. As a result, the serial correlation of tax rates falls between those of the government expenditure process and the government debt process.

4.2 Discussions

In the representative-agent economy of Aiyagari et al. (2002), a high level of household debt (and hence government asset) helps the government reduce taxes in the future. Lower taxes, in turn, boost production and consumption. Thus, for the representative household, borrowing does not come at the expense of less consumption in the future. This is how the precautionary saving motive of the household is subsumed into the optimal fiscal policy.

In an economy with undiversifiable idiosyncratic income risk, however, the link between individual households' borrowing and low tax rates is severed. For individual households, a high level of debt will lower their consumption in the future, because future tax rates are determined by the government debt level, not by the debt level of one particular household. The presence of idiosyncratic risk not insured by the tax policy brings out an independent role for the self-insurance of the households.

As indicated above, whether the government should build up a war chest of assets or issue debt to the private sector is determined by the relative strengths of their precautionary saving motives. The intensity of these precautionary saving motives are, in turn, determined by the magnitude and persistence of the aggregate versus idiosyncratic risk. There are empirical evidences consistent with this prediction. Mendoza and Oviedo (2006) show that there is a strong negative correlation between countries' aggregate volatility and their indebtedness.

To be more precise, what matters is the magnitude of the risk uninsurable by other means than risk-free consumption loans. For example, if the individual risk can be fully diversified away, the model collapses into the representative agent model of Aiyagari et al. and the optimal fiscal policy will be invariant to the magnitude of idiosyncratic risk. The current model abstracts away from the instruments that diversify and reallocate idiosyncratic risk in reality (e.g. disability and unemployment insurance). In this light, the idiosyncratic risk in the model should be interpreted as "residual" risk that are left uncovered by all other means of insurance.

According to the latest *OECD Economic Outlook*, No. 79 (Annex Table 33), the net public financial liabilities (2006 figures as per cent of 2006 nominal GDP) of Scandinavian countries are either *negative* (Finland: -61.1; Norway: -146.3; Sweden: -17.6) or very low (Denmark: 3.8) compared to the OECD average of 46.9. Given the strongly egalitarian nature of these countries' taxation and redistribution policies, this observation is compatible with the model's prediction.

In a similar vein, the model makes a prediction about the relationship between progressivity in taxation and government debt. Progressive taxation provides more insurance against idiosyncratic risk than flat-rate taxes do (Vickrey, 1947). Other things being equal, a move away from progressive taxation to flat-rate taxes will generate more residual idiosyncratic risk, and hence increase the optimal level of government debt.

Another factor affecting the optimal level of public debt is how much private IOUs (or "inside money") can be issued by households. The current model grants a very lenient borrowing limit to the households (natural debt limit). A tighter borrowing constraint leads to less supply of private IOUs and more demand for precautionary saving (Aiyagari, 1994;

Huggett, 1993). Therefore, the equilibrium level of government debt will be higher.

For these reasons, any serious attempt at calculating the optimal level of government debt must be based on reliable measurement of residual idiosyncratic risk and households' borrowing limit, as well as the properties of the aggregate shock. An immediate policy implication is that there is no uniformly optimal level of government debt (or assets), in that countries differ from one another in terms of idiosyncratic and aggregate risk, private and public insurance provisions, private sector's ability to issue IOUs, progressivity of taxes and so on.

5 Concluding Remarks

Dynamic optimal taxation problems in the Ramsey tradition have a common structure. The model builder exogenously specifies restrictions on the tax instruments available to the government (e.g., they have to be flat-rate taxes) and, in the emerging incomplete markets branch of the literature, some additional restrictions on the range of assets that can be traded by households and the government. The Ramsey planner calculates a tax and debt policy that copes with these restrictions. A given set of restrictions on markets and fiscal instruments constrains the planner even more when, as in this paper, the model builder increases the diverse sources of risk that he confronts. The incompleteness of markets for sharing risk motivates both households and the government to self-insure by accumulating assets. As usual, the self-insurance motive of households are encoded in the implementability constraints faced by the Ramsey planner. This paper has described a sense and a setting in which there are separate precautionary saving motives attributable to the government and to households.

I have shown that idiosyncratic elements of the economy—undiversifiable individual income risk and households' self-insurance, as well as aggregate risk—determine the optimal financing decision of the government with incomplete markets. As emphasized in the earlier sections, adding such elements brings theoretical outcomes closer to observed outcomes in one important respect—the mean level of government debt becomes positive.

An ensuing question is how the introduction of such idiosyncratic heterogeneity will affect the results of the existing optimal policy literature built on representative-agent models. Answering this question in the context of optimal capital taxation (Chari et al., 1994), optimal monetary policy (Chari et al., 1991; Schmitt-Grohé and Uribe, 2004; Siu, 2004) and optimal maturity structure of government debt (Angeletos, 2002; Buera and Nicolini, 2004) is left for future research.

Appendix A Two-Period Economy

The Ramsey problem is to maximize the objective function:

$$\mathbb{E}_0 \sum_{t=0}^{1} \sum_{i=1}^{2} \left(\log c_t^i + \log x_t^i \right),$$

subject to household budget constraints (HB), government budget constraints, resource constraints (RC), asset market clearing conditions, and implementability conditions (IM) in all periods and states. In this closed economy, the net private savings carried over to $t = 1$ $(a_1^1 + a_1^2)$ must be equal to the quantity of government debt issued at $t = 0$ (b_1). The government budget constraints become redundant once household budget constraints and resource constraints are satisfied, and are omitted in the analysis below.

First, I solve for the allocations at $t = 1$, after the resolution of uncertainty. Let a_1^1 and a_1^2 respectively denote the asset holdings of the two types of households at the beginning of $t = 1$. Given the ex ante symmetry between the two types, I can impose $a_1^1 = a_1^2$. At $t = 1$, conditioning on $a_1 (= a_1^1 = a_1^2)$ and the shock realization, the problem is to maximize:

$$\log c_{1h} + \log x_{1h} + \log c_{1l} + \log x_{1l},$$

subject to:

$$\begin{aligned}
\text{RC}: &\quad c_{1h} + c_{1l} + g_1 = \theta_{1h}(1 - x_{1h}) + \theta_{1l}(1 - x_{1l}),\\
\text{HB}: &\quad c_{1h} - a_1 = (1 - \tau_1)\theta_{1h}(1 - x_{1h}), \quad c_{1l} - a_1 = (1 - \tau_1)\theta_{1l}(1 - x_{1l}),\\
\text{IM}: &\quad \frac{c_{1h}}{\theta_{1h} x_{1h}} = 1 - \tau_1 = \frac{c_{1l}}{\theta_{1l} x_{1l}}.
\end{aligned}$$

The subscripts h and l respectively refer to the households with $\theta_{1h} = 1 + \sigma$ and $\theta_{1l} = 1 - \sigma$. For notational convenience, let $y_{1h} = (1 + \sigma)(1 - x_{1h})$ and $y_{1l} = (1 - \sigma)(1 - x_{1l})$, with $y_1 = y_{1h} + y_{1l}$. The solution to the $t = 1$ problem can be written as follows.

$$y_1 = \frac{1 + g_1 + a_1 + \sqrt{(1 - g_1 - a_1)^2 - 4a_1}}{2} \tag{8}$$

$$c_{1h} = \frac{(y_1 - g_1)(2 + \sigma - y_1)}{4 - 2y_1}, \quad c_{1l} = \frac{(y_1 - g_1)(2 - \sigma - y_1)}{4 - 2y_1} \tag{9}$$

$$x_{1h} = \frac{2 + \sigma - y_1}{2 + 2\sigma}, \quad x_{1l} = \frac{2 - \sigma - y_1}{2 - 2\sigma} \tag{10}$$

Solving backwards, the problem at $t = 0$ can be posed as the maximization of:

$$W(a_1) = 2\log c_0 + 2\log x_0 + 2\mathbb{E}\left[\log \frac{y_1 - g_1}{2 - y_1} + \log\left(2 + \sigma - y_1\right)(2 - \sigma - y_1)\right] + C(\sigma),$$

subject to the following constraints in the initial period:

$$\text{RC}: \quad 2c_0 + g_0 = 2(1 - x_0),$$

$$\text{HB}: \qquad c_0 + a_1 c_0 \mathbb{E}\left[\frac{1}{c_1}\right] = \frac{c_0}{x_0}(1 - x_0).$$

Ex ante symmetry across households implies that $c_0^1 = c_0^2$ and $x_0^1 = x_0^2$. $C(\sigma)$ is a collection of terms that are not affected by a_1. The prices at $t = 0$ have been substituted out by individual optimality conditions. The price of the risk-free bond at $t = 0$ is replaced by $c_0 \mathbb{E}\frac{1}{c_1}$, and the tax rate by $1 - \frac{c_0}{x_0}$. As shown in (8), y_1 is a function of a_1 and g_1. In the following analysis, the dependence of y_1 on g_1 is indicated by subscripts L and H, which correspond to g_L and g_H. Equation (8) requires $(1 - g_1 - a_1)^2 - 4a_1 \geq 0$, which holds when $a_1 = 0$.

Appendix B Proof of Proposition 1

When neither σ nor Δ is zero, differentiating the welfare with regard to a_1 yields:

$$\left(\frac{2}{x_0} - \frac{2}{c_0}\right)\frac{\partial x_0}{\partial a_1} + \left(\frac{1}{y_{1L} + \Delta} + \frac{1}{2 - y_{1L}} - \frac{1}{2 + \sigma - y_{1L}} - \frac{1}{2 - \sigma - y_{1L}}\right)\frac{\partial y_{1L}}{\partial a_1}$$
$$+ \left(\frac{1}{y_{1H} - \Delta} + \frac{1}{2 - y_{1H}} - \frac{1}{2 + \sigma - y_{1H}} - \frac{1}{2 - \sigma - y_{1H}}\right)\frac{\partial y_{1H}}{\partial a_1}, \tag{11}$$

where I used $\frac{\partial c_0}{\partial a_1} = -\frac{\partial x_0}{\partial a_1}$. When $a_1 = 0$, $x_0 = c_0 = \frac{1}{2}$ and $y_{1L} = y_{1H} = 1$. Evaluated at $a_1 = 0$, the formula turns into:

$$-\left(\frac{1}{1+\Delta} + 1 - \frac{1}{1+\sigma} - \frac{1}{1-\sigma}\right)\frac{1}{1+\Delta} - \left(\frac{1}{1-\Delta} + 1 - \frac{1}{1+\sigma} - \frac{1}{1-\sigma}\right)\frac{1}{1-\Delta}$$
$$= \frac{4}{1-\Delta^2}\left(\frac{1}{1-\sigma^2} - \frac{1}{1-\Delta^2}\right).$$

Therefore, if $\sigma > \Delta$ ($\sigma < \Delta$), $\frac{\partial W}{\partial a_1}|_{a_1=0}$ is greater (less) than zero, validating the first two claims of the proposition.

It follows that $\sigma = \Delta$ implies $\frac{\partial W}{\partial a_1}|_{a_1=0} = 0$. The second derivative of the welfare with respect to a_1 is:

$$\underbrace{\left(\frac{2}{x_0} - \frac{2}{c_0}\right)\frac{\partial^2 x_0}{\partial a_1^2} - \left(\frac{2}{x_0^2} + \frac{2}{c_0^2}\right)\left(\frac{\partial x_0}{\partial a_1}\right)^2}_{Q_1}$$
$$+ \underbrace{\left(\frac{1}{y_{1L} + \sigma} + \frac{1}{2 - y_{1L}} - \frac{4 - 2y_{1L}}{(2 - y_{1L})^2 - \sigma^2}\right)\frac{\partial^2 y_{1L}}{\partial a_1^2} + \left(\frac{1}{y_{1H} - \sigma} + \frac{1}{2 - y_{1H}} - \frac{4 - 2y_{1H}}{(2 - y_{1H})^2 - \sigma^2}\right)\frac{\partial^2 y_{1H}}{\partial a_1^2}}_{Q_2}$$
$$+ \underbrace{\left(\frac{1}{(2 - y_{1L})^2} - \frac{1}{(y_{1L} + \sigma)^2} - \frac{1}{(2 + \sigma - y_{1L})^2} - \frac{1}{(2 - \sigma - y_{1L})^2}\right)\left(\frac{\partial y_{1L}}{\partial a_1}\right)^2}_{Q_3}$$
$$+ \underbrace{\left(\frac{1}{(2 - y_{1H})^2} - \frac{1}{(y_{1H} - \sigma)^2} - \frac{1}{(2 + \sigma - y_{1H})^2} - \frac{1}{(2 - \sigma - y_{1H})^2}\right)\left(\frac{\partial y_{1H}}{\partial a_1}\right)^2}_{Q_4}. \tag{12}$$

The Q's can be evaluated at $a_1 = 0$ one by one.

$$Q_1|_{a_1=0} = -16\left(\frac{\partial x_0}{\partial a_1}\right)^2 \leq 0$$

$$Q_3|_{a_1=0} = \left(1 - \frac{2}{(1+\sigma)^2} - \frac{1}{(1-\sigma)^2}\right)\frac{1}{(1+\sigma)^2} < 0$$

$$Q_4|_{a_1=0} = \left(1 - \frac{1}{(1+\sigma)^2} - \frac{2}{(1-\sigma)^2}\right)\frac{1}{(1-\sigma)^2} < 0$$

To determine the sign of $Q_2|_{a_1=0}$, first derive $\frac{\partial^2 y_1}{\partial a_1^2}$.

$$\left.\frac{\partial^2 y_1}{\partial a_1^2}\right|_{a_1=0} = \frac{1}{2}\left(\frac{1}{\sqrt{(1-g_1-a_1)^2-4a_1}} - \frac{(a_1+g_1-3)^2}{[(1-g_1-a_1)^2-4a_1]^{\frac{3}{2}}}\right)\Bigg|_{a_1=0} = \frac{2g_1-4}{(1-g_1)^3}$$

Then $Q_2|_{a_1=0}$ becomes:

$$\frac{2\sigma}{1-\sigma^2}\frac{\sigma+2}{(1+\sigma)^2} + \frac{2\sigma}{1-\sigma^2}\frac{\sigma-2}{(1-\sigma)^2} = \frac{4\sigma^2(\sigma^2-3)}{(1-\sigma^2)(1+\sigma)^2(1-\sigma)^2} \leq 0,$$

because the relevant range of σ is $[0,1)$. Therefore, $\frac{\partial^2 W}{\partial a_1^2}|_{a_1=0} < 0$, and $a_1 = 0$ attains local maximum when $\sigma = \Delta$. ∎

Appendix C Recursive Formulation

As in Kydland and Prescott (1980), the system is not recursive in the "natural" state variables $(a_t^1, a_t^2; s_{t-1})$, but becomes recursive if the state space is expanded to include $(u_{c,t-1}^1, u_{c,t-1}^2)$. It turns out that the dimension of the state space can be reduced by embedding all the information in $(\alpha_t^1, \alpha_t^2, \rho_{t-1}; s_{t-1})$, where $\alpha_t^i = a_t^i \mathbb{E}_{t-1}[u_{c,t}^i]$ and $\rho_{t-1} = \frac{u_{c,t-1}^1}{u_{c,t-1}^2}$. This choice of state space is a variation of Werning (2003)'s approach. The timing of the recursive problem is at the beginning of each period, but before the current shock realization is known.

$$W(\alpha_1, \alpha_2, \rho_-; s_-) = \max\ \mathbb{E}\left[\sum_{i=1}^{2} u(c_i(s), x_i(s)) + \beta W\left(\alpha_1'(s), \alpha_2'(s), \frac{u_{c1}(s)}{u_{c2}(s)}; s\right)\right]$$

$$s.t. \quad c_1(s) + c_2(s) + g(s) = \theta_1(s)(\xi - x_1(s)) + \theta_2(s)(\xi - x_2(s)),\ \forall s \tag{13}$$

$$c_1(s) + \beta\frac{\alpha_1'(s)}{u_{c1}(s)} = (1-\tau(s))\theta_1(s)(\xi - x_1(s)) + \frac{\alpha_1}{\eta_1},\ \forall s \tag{14}$$

$$c_2(s) + \beta\frac{\alpha_2'(s)}{u_{c2}(s)} = (1-\tau(s))\theta_2(s)(\xi - x_2(s)) + \frac{\alpha_2}{\eta_2},\ \forall s \tag{15}$$

$$\frac{u_{x1}(s)}{\theta_1(s)u_{c1}(s)} = 1 - \tau(s) = \frac{u_{x2}(s)}{\theta_2(s)u_{c2}(s)},\ \forall s \tag{16}$$

$$\eta_1 = \mathbb{E}u_{c1},\ \eta_2 = \mathbb{E}u_{c2},\ \frac{\eta_1}{\eta_2} \gtreqless \rho_- \tag{17}$$

The maximization is over (η_1, η_2) and $\{c_1(s), c_2(s), x_1(s), x_2(s), \alpha_1'(s), \alpha_2'(s), \tau(s)\}$ for all s. The constraints (13) through (16) must be satisfied for all s. $\frac{\eta_1}{\eta_2} = \rho_-$ will hold when the households' borrowing constraints do not bind. If type-1 (type-2) households' borrowing limit is binding, then $\frac{\eta_1}{\eta_2} < \rho_-$ $\left(\frac{\eta_1}{\eta_2} > \rho_-\right)$. Once the constraints are accounted for, there are $2N_s - 1$ free variables, where N_s is the number of possible realizations of s. I substitute out $x_1(s)$, $x_2(s)$, $\tau(s)$ by (13) and (16); $\alpha_1'(s)$, $\alpha_2'(s)$ by (14) and (15). The maximization in practice is over $c_1(s)$, $\rho(s)$ subject to (17), with $c_2(s)$ being determined concurrently. The regions with binding debt limits are treated separately.

Appendix D Numerical Implementation

The presence of three continuous state variables $(\alpha_1, \alpha_2, \rho_-)$ and the need to optimize over $2N_s - 1$ variables at every node demand substantial computing power and time. In spite of the nonconvexities, all the numerical value function iterations converged monotonically.

1. Start with a state space that is "big enough". Let $\mathcal{A} = \{\bar{\alpha}_1, \ldots, \bar{\alpha}_N\}$ and $\mathcal{R} = \{\bar{\rho}_1, \ldots, \bar{\rho}_M\}$. Construct $\mathcal{R}$ in such a way that $\bar{\rho}_m \in \mathcal{R}$ implies $\frac{1}{\bar{\rho}_m} \in \mathcal{R}$ for all m. Then, the symmetry of $W(\alpha_1, \alpha_2, \rho; \theta_1, \theta_2, g) = W(\alpha_2, \alpha_1, \frac{1}{\rho}; \theta_2, \theta_1, g)$ can be exploited to halve the computing time. Let ω denote a typical element of $\mathcal{A}^2 \times \mathcal{R}$. The results in Section 4.1 are computed with $N = 126$ and $M = 251$.
2. Make an initial guess on the $N^2 \times M$ elements of $W^0(\omega; s)$ for each s. Because the one-period return function cannot be bounded from below, it is imperative that $\min_{\omega \in \mathcal{A}^2 \times \mathcal{R}} \left(W^0(\omega; s) - W(\omega; s)\right) \geq 0$ for each s. One candidate W^0 can be computed under the assumption that there is no uncertainty in the economy, with $(\theta_t^1, \theta_t^2, g_t) = (1, 1, g_L)$ for all t.
3. At each $(\omega; s)$, maximize the right-hand side of the Bellman equation, subject to the constraints (13)–(17). I use a direct-search polytope algorithm for the maximization, after all the equality constraints are substituted in. To evaluate W^0 off the grid, I use tri-linear interpolation on the grid cube—which is essentially the three-dimension analogue of piece-wise linear interpolation. Given the nonconvexities in the problem, this simple interpolation scheme seems to be more robust than other tensor-product splines of higher order.
4. Construct W^1 from the maximized values at each $(\omega; s)$. Repeat Step 3, replacing W^0 with W^1. Iterate until convergence.
5. Simulate off sample paths under the optimal decision rule, and compute sample moments of interest.
6. Enlarge the state space, and redo Steps 2 through 5. Repeat until the computed sample moments do not change.

References

AIYAGARI, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109, 659–684.

AIYAGARI, S. R., A. MARCET, T. J. SARGENT, AND J. SEPPÄLÄ (2002): "Optimal Taxation without State-Contingent Debt," *Journal of Political Economy*, 110, 1220–1254.

AIYAGARI, S. R. AND E. R. MCGRATTAN (1998): "The Optimum Quantity of Debt," *Journal of Monetary Economics*, 42, 447–469.

ANGELETOS, G.-M. (2002): "Fiscal Policy with Non-Contingent Debt and the Optimal Maturity Structure," *Quarterly Journal of Economics*, 117, 1105–1131.

BARRO, R. J. (1979): "On the Determination of Public Debt," *Journal of Political Economy*, 87, 940–971.

BEWLEY, T. F. (1980): "The Optimum Quantity of Money," in *Models of Monetary Economies*, ed. by J. H. Kareken and N. Wallace, Federal Reserve Bank of Minneapolis, 169–210.

——— (1983): "A Difficulty with the Optimum Quantity of Money," *Econometrica*, 51, 1485–1504.

BUERA, F. J. AND J. P. NICOLINI (2004): "Optimal Maturity of Government Debt without State Contingent Bonds," *Journal of Monetary Economics*, 51, 531–554.

CHARI, V. V., L. J. CHRISTIANO, AND P. J. KEHOE (1991): "Optimal Fiscal and Monetary Policy: Some Recent Results," *Journal of Money, Credit and Banking*, 23, 519–539.

——— (1994): "Optimal Fiscal Policy in a Business Cycle Model," *Journal of Political Economy*, 102, 617–652.

——— (1995): "Policy Analysis in Business Cycle Models," in *Frontiers of Business Cycle Research*, ed. by T. F. Cooley, Princeton: Princeton University Press, 357–391.

HALL, R. E. (1978): "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, 86, 971–987.

HOLMSTRÖM, B. AND J. TIROLE (1998): "Private and Public Supply of Liquidity," *Journal of Political Economy*, 106, 1–40.

HUGGETT, M. (1993): "The Risk-Free Rate in Heterogeneous-Agent Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control*, 17, 953–969.

Kydland, F. E. and E. C. Prescott (1980): "Dynamic Optimal Taxation, Rational Expectations and Optimal Control," *Journal of Economic Dynamics and Control*, 2, 79–91.

Lucas, Jr., R. E. and N. L. Stokey (1983): "Optimal Fiscal and Monetary Policy in an Economy without Capital," *Journal of Monetary Economics*, 12, 55–93.

Mendoza, E. G. and P. M. Oviedo (2006): "Fiscal Policy and Macroeconomic Uncertainty in Developing Countries: The Tale of the Tormented Insurer," Manuscript, University or Maryland.

Ramsey, F. P. (1927): "A Contribution to the Theory of Taxation," *Economic Journal*, 37, 47–61.

Schmitt-Grohé, S. and M. Uribe (2004): "Optimal Fiscal and Monetary Policy under Sticky Prices," *Journal of Economic Theory*, 114, 198–230.

Siu, H. E. (2004): "Optimal Fiscal and Monetary Policy with Sticky Prices," *Journal of Monetary Economics*, 51, 575–607.

Vickrey, W. S. (1947): *Agenda for Progressive Taxation*, New York: Ronald Press, first ed.

Werning, I. (2003): "Standard Dynamic Programming for Aiyagari, Marcet, Sargent and Seppälä (2002)," Unpublished lecture note, Department of Economics, Massachusetts Institute of Technology.